L'ELETTROTECNICA

GIORNALE ED ATTI DELLA

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CONTENT

NOTE DI REDAZIONE

I nostri lettori osserveranno subito che questo numero del nostro giornale presenta una fisionomia del tutto particolare, sia nella forma sia nel contenuto: esso è infatti compilato interamente in lingua inglese e contiene un solo scritto, senza le ordinarie rubriche.

Il fascicolo è destinato unicamente a rendere noto uno scritto rimasto inedito, e in qualche punto incompleto, di Giuseppe Massimo Pestarini, il cui nome, noto a tutti gli elettrotecnici italiani e stranieri, è indissolubilmente legato a quella interessantissima categoria di macchine che si ricollegano alla metadinamo.

Si è voluto con questo fascicolo del nostro giornale, rendere omaggio alla memoria del grande Scomparso che ha altamente onorato la tecnica italiana; e, nello stesso tempo, impedire che restasse ignorato l'ultimo Suo lavoro al quale aveva dedicato l'attività degli ultimi anni, e che, trattando della dinamica della metadinamo, completa insieme al volume già noto sulla statica della metadinamo, lo studio di questa macchina da parte del Suo inventore.

L'iniziativa del Presidente Generale, prof. Angelini, di far conoscere questo lavoro del compianto prof. Pestarini sarà certamente assai bene accolta da tutti e ben si addice all'indole del nostro giornale che molte volte ha accolto gli scritti fondamentali dei maggiori elettrotecnici italiani, come, per citare solo pochi esempi, quelli di Giorgi e di Allievi. E non è escluso che il presente numero speciale abbia ad essere seguito in futuro da altri analoghi fascicoli.

Aggiungiamo che è intenzione della Presidenza di curare successivamente anche una traduzione italiana del presente scritto, sotto forma di una « Monografia ».

Il manoscritto del presente lavoro era stato dall'autore compilato in lingua inglese e si è ritenuto quindi opportuno pubblicarlo nella forma originale, anche per dare ad esso una maggiore diffusione in campo internazionale; tanto più che anche il volume sulla statica della metadinamo è stato, a suo tempo, pubblicato in inglese.

Il prof. Pestarini non ebbe la possibilità di completare il Suo scritto e tanto meno di rivederlo e riordinarlo. Il manoscritto richiedeva, per la pubblicazione, un lavoro assai delicato, paziente e difficile, di revisione, di riordino, in qualche punto anche di completamento.

Un simile compito, non privo di grave responsabilità, richiedeva, in chi si accingesse ad eseguirlo, una profonda competenza, una dedizione paziente e disinteressata, una fatica pesante anche se apportatrice di intima soddisfazione.

Fu ventura che si trovasse fra gli studiosi italiani chi non esitò ad assumersi un tale incarico e seppe condurlo a termine con pieno successo. I tecnici italiani devono essere grati al prof. Alfredo Vallini che si è dedicato a questo compito con la passione, e lo spirito di sacrificio che solo l'entusiasmo e la lunga consuetudine di studio possono donare.

La Redazione de « L'Elettrotecnica » è lieta di interpretare il sentimento di tutti i membri dell'AEI per porgere un vivo ringraziamento al prof. Vallini.

LA REDAZIONE

IN MEMORY OF GIUSEPPE MASSIMO PESTARINI

ARNALDO M. ANGELINI (*)

In his introduction to the volume, « Metadyne Statics », written by the late Giuseppe Pestarini at the time when he was teaching at the Massachusetts Institute of Technology, Professor Harold L. Hazen, a colleague of his at the world famous university, noted that the book offered a solid foundation for a successive investigation of the dynamics of the metadynamo, with particular reference to the field of automatic control systems. Professor Pestarini had, in effect, written a considerable part of this « successive investigation » when he passed away in the summer of 1957.

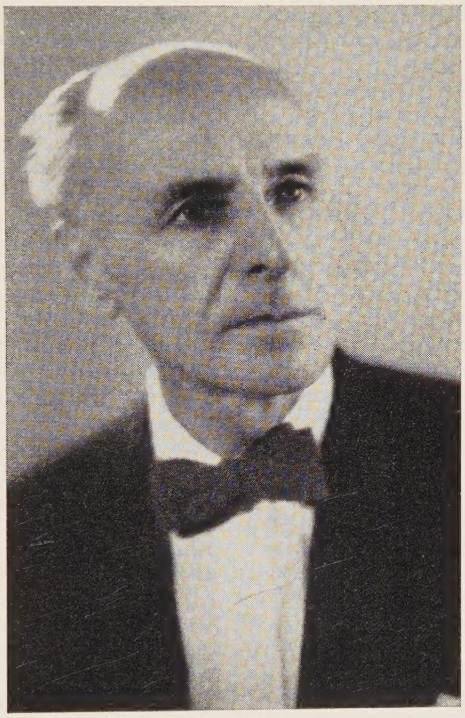
Parts of the unfinished manuscript had, moreover, served as the basis for lectures and for conferences in Italy, in the United States, in Switzerland, in Sweden, and probably elsewhere.

It seems to me that the publication of these manuscripts represents the best and most fitting way of re-evocating, in these pages (and in a monograph which will be published in its original English text and in Italian) the stature of our late friend and distinguished colleague. He was a man who contributed greatly, in theoretical, experimental, and applied fields, to the progress of electrotechnical engineering in general and to that of electrical machinery in particular.

Grateful for the unanimous support which this initiative received from our Association, from the National Council of Research, the National Electrotechnical Institute Galileo Ferraris of Turin, and the Department of Engineering at the University of Rome, I would like to thank Mrs Pestarini for having gathered together her late husband's manuscripts with such care and for having kindly placed them at my disposal.

I also wish to thank Professor Vallini individually. A former student of our late colleague, he willingly undertook the delicate task of revisioning and organizing these manuscripts, thus giving thousands of other men working in this field the chance to examine a wealth of ideas and theories which will unquestionably prove significant.

The singular zeal with which prof. Vallini dedicated time and effort to this work, which he accomplished with unexpected energy and skillful competence, should earn him the deepest and sincerest praise. But for him this great work of the Maestro might never have come to be published. Prof. Vallini virtually acted as an author in that final revision which premature death prevented the actual author from undertaking.



It would be picayune, I think, to talk of Pestarini's work in a purely chronological sense. Instead, I should like to recall his brilliance, his influence as a man of rare intellectual gifts, and the importance of his work to the development of the electrotechnical field.

Without recourse to conventional expressions of condolence which would be out of place here, I would like to emphasize, above all, that Pestarini was a profoundly good man.

This rare command of goodness was the source of an intelligent optimism which never failed him, even in the black crisis of the second world war. Pestarini's calm and serenity during those tragic years had a deeply positive effect on his friends, colleagues, and students.

His truly exceptional talents, were, in many ways, manifestations of a singular personality. It is not surprising, therefore, that he applied his intelligence to a

(*) by Arnaldo Maria Angelini, Professor at the University of Rome.

wide number of fields outside his specialization; few of his colleagues are aware, probably, of his passion for art and profound knowledge of design and painting.

His grasp of physics and mathematics was vast and particularly brilliant in certain sectors. The language of mathematics came to him almost effortlessly, and he used it with remarkable facility. I should add that mathematics was only one of the languages at his command, since he was also perfectly fluent in Italian, French, English, German and Greek.

Whoever attended his lectures, read his books or knew the pleasure of his conversation, soon became aware that the studies he vent through at the Athens Polytechnic, at l'Ecole Superieur d'Electricité de Paris and at the Department of Mathematical Sciences at Pavia, were for him only the base for later on rehandling and developing in an entirely personal manner the knowledge he had acquired.

If, in fact, his writings are occasionally difficult to follow, it is due to the highly personal way he had of approaching ideas.

Utilizing his powerful analytical ability, he was naturally inspired to produce large and often general solutions, which he almost always sought along original lines of thought, far more preoccupied with having his audiences and readers participate in his original approach to a subject than in following the conventional methods already indicated by his predecessors.

He once said to me, in fact, that he preferred dedicating his time to generalizing and exposing the scientific fundamentals of the electrotechnical and electromechanical fields according to his personal vision than to elaborating the work of other men.

It is hardly surprising, therefore, that so many original contributions should bear his signature; the most important, in a pratical sense, being his invention of the metadynamo. As another late authority, Professor Giovanni Giorgi, once said, « when for decades one had heard that the epoch of electromechanical inventions was already behind us, Giuseppe Pestarini surprised electrical engineers the world over with his metadynamo, a machine which has proved to have a great many uses » (*).

The practical importance of this invention is too well known to require further exposition. I would, nevertheless, like to include part of Blondel's introduction to '' Les Metadynes '': « Ce travail constitue un chapitre nouveau et important de la théorie des machines à courant continu et fait grand honneur à son auteur ». And Boucherot, in a personal letter to Pestarini dated 1937, wrote: « Je connaissais vos travaux théoriques sur ce sujet puisque, en particulier, je fus du Jury Montefiore à qui vous les soumites. Mais je déplorais, il y a peu de temps encore, devant mes élèves, que cela ne sorte pas du laboratoire. C'est fait maintenant, et je m'en réjouis: Voici enfin que l'on s'engage dans cette voie, que je préconise depuis plus de trente ans, de l'emploi de transformations de distributions à V constante en distributions à I constante, pour les appareils de manutention à courant continu. C'est vraiment du nouveau. Et cela, je le dis encore très sincèrement, par un procédé supérieur à ceux que j'ai imaginés dans ce but, il y a longtemps, si vous avez réussi à vaincre les difficultés de la commutation ».

This invention, which has been defined as « the gateway to an expanding field of electrical applications », developed from the original research done by Pestarini on commutation and incorporates, as is well known, a generalization of the continuous current dynamo which, under the name of metadynamo, includes a totally new category of machines able to perform a great number of functions.

The references to the invention of the metadynamo in the technical journals the world over were significant.

Metadynamos have been used extensively in the control rooms of ships and in the controls of rolling mills, in the control systems of large dynamos employed in chemical and other industrial plants, in electric and Diesel electric locomotives; (the Illinois Central Railroad has widely employed metadynamos in plants of the latter type).

It is probable that the possibilities opened up by Pestarini's invention will soon be exploited in the vast field of automatic controls.

As strange as it may seem, new applications in the expanding category of continuous current machines grouped under the name of metadynamo would have been far more rapid if the great variety of its characteristics and, therefore, of electrical and construction schemes, had not been so complex, forcing contractors and consumers to undergo an arduous period of specialization.

The fact that the metadynamo is Pestarini's best known invention should not be allowed to osbscure the many important contributions he made to other electrotechnical fields, investigating a number of particularly difficult problems which he then illuminated and analyzed.

The index of his publications is a telling indication of his professional scope.

Among these works I should like to point to his penetrating and important study of ponderous motor forces acting on electric and magnetic circuits, and, in particular, on the applications of the general theories of potential electromagnetic and electrostatic energy (a very original piece of research); to the determination of the tresses in circuits and the means by which they are organized; to the study and determination of central forces acting on the rotors of asynchronous motors, and on the effects of the decentralization of the rotor in electrical machinery.

Other investigations of his concerned the wide field of magnetizing powers and moments. Pestarini did a great deal of work in this field, developing a series of new theorems which he applied to a wide range of individual problems.

Various articles dealing with these subjects were published in « L'Elettrotecnica »; others appeared in foreign reviews, particularly in France.

During the years he spent in Italy, where he was Professor of Electrical Machinery at the National Electrotechnical Institute Galileo Ferraris of Turin, and later a member of the Department of Engineering at the University of Rome, he prepared a selection of notes for the use of his students and dedicated himself to the publication of a work he entitled « Elettromeccanica »; Only the first volume appeared, containing the fundamentals of general electrotechnical engineering.

His transfer to the United States in 1947 interrupted the writing of subsequent volumes. In the one he published, as in all his writings, Pestarini was characteristically concerned with the organization and elegance of his exposition, summarizing the results of his theoretical research with the originality of approach I have already mentioned.

It is certainly no exageration to affirm that the bulk of his work represents a true bounty which young engineers can use as a basis for their own thinking and explorations.

In the note which concludes the above-cited volume, Pestarini himself, referring to the contents of the book, states that « new subjects must be presented to the public supported by detailed proofs, so as not to invoke doubts of any sort. Only after other experts have examined and elaborated them will they have been rendered more elegant and more concise ».

And since I have the volume here beside me as I write this, let me include a section of his preface in which he

(*) « Il Giornale delle Scienze » - July 25, 1957.

indicates the structure of his method and the approach he liked to assume towards his favorite type of problems:

« The author who is asked to apply himself almost continually to unusual problems of electricity, of calculation and design, knows that in the majority of cases the conventional approach will lead him only to failure; he knows that he is guided to the solution largely by means of intuitive reasoning, the result of a privileged instinct which conforms to reality, an instinct which must be regarded as an intellectual gift but which becomes effective only after a long preparation in rigorous analysis sustained by continuous practice; what one asks, after all, regarding practical problems, is that they be resolved successfully, not the reasons underlying the success ».

I have barely touched on Pestarini's contributions to industry and to teaching; let me therefore indicate the principal milestones of a brilliant career.

Having finished his studies at l'Ecole Supérieure d'Electricité, Pestarini was recommended by one of his teachers, Janet, to the Thompson-Houston Company in Paris, where he was employed in 1911 and where he rapidly became the chief engineer.

During the first world war, he joined the Italian Westinghouse Co. as head engineer and was successively appointed consultant to the English company, Metropolitan Wickers.

After the publication of his studies on the metadynamo, he was offered posts as consultant to the General Electric Company, the Allgemeine Elektricitats Gesellschaft of Berlin, the A.S.E.A. of Vasteras (Sweden), the San Giorgio of Genova, and many other large firms constructors electrical machinery.

While working at the San Giorgio, he was invited by Professor Vallauri, whom he had met during a brief period of teaching at the Livorno Academy, to lecture on the « Construction of Electrical Machinery » at the Turin Polytechnic. In 1937, having placed first in the competition, he received his professorship from that institution, later transferring to the University of Rome as a professor in the same field.

The last years of his noble and laborious existence were spent in the United States, where he was a well known and highly reputed teacher at the Massachusetts Institute of Technology, at Columbia University, at the University of California, and served as a professor at the University of Minnesota. In addition to this intense scholastic activity, he was also consultant to a number of important industrial firms.

It was, therefore, in the midst of academic and scientific labours that he passed away, leaving incomplete the work which his friends and admirers present in the following pages, a work which will be added to the distinguished list reproduced on page 830.

The Publications of Giuseppe Massimo Pestarini

La mesure des puissances réelles et réactives dans les réseaux polyphasés. « Bulletin de la Société Internationale des Electriciens », marzo 1914, Tomo IV, 3e serie, 33, p. 273.

Méthode pour la détermination des transformateurs. « La Lumière Electrique », 8 gennaio 1916, Tomo XXXII, 2e serie, 2, p. 29.

Considérations et diagrammes sur le montage. Scott appliqué aux transformateurs et aux machines tournantes. « La Lumière Electrique », 5 febbraio 1916, Tomo XXXII, 2e serie, 6, p. 125.

Apparatus for measuring very strong or very weak alternating currents, with adjustable field magnet. « Electrician », 2, novembre 1917, p. 1.

Breve memoria sulla parallelizzazione degli statori, dei motori delle locomotive trifasi. « L'Elettrotecnica », 15 marzo 1918, V. 8, p. 102.

Circuiti ad oscillazioni permanenti. « L'Elettrotecnica », 15 agosto 1919, VI, 23, p. 481.

Contributo al problema del motore a induzione a più velocità di sincronismo. « L'Elettrotecnica », 5 gennaio, 25 gennaio 1922, IX, 1, 2, 3, pp. 8, 27, 49.

Trasformatrici polimorfiche. « L'Elettrotecnica », 5, 15 dicembre 1922, IX, 34, 35, pp. 777, 797.

Une butée magnétique. « Electricité et Mécanique », luglio-agosto 1927, 19, p. 1.

Sur la théorie des puissances complexes. « Electricité et Mécanique », maggio-giugno 1928, 24, p. 1.

Sur la théorie des puissances complexes (2me partie). « Electricité et Mécanique », luglio-agosto 1928, 25, p. 1.

Sur la théorie des puissances complexes (3me partie). « Revue d'Electricité et de Mécanique », sett.-ott. 1928, 1, p. 42.

Les forces centrales, dans les moteurs asynchrones. « Revue Générale de l'Electricité », 6 ottobre 1928. Tomo XXIV, 14, p. 517.

Della Metadinamo. « L'Elettrotecnica », 15 novembre 1928, XV, 32, p. 853.

Appendice au mémoire sur la théorie des puissances complexes. « Revue d'Electricité et de Mécanique », gennaio-febbraio 1929, 3, é. 1.

Sur un genre particulier d'Ampèremètre. « Revue d'Electricité et de Mécanique », maggio-giugno 1929, 5, p. 12.

Première contribution au problème de la commutation. « Revue Générale de l'Eléctricité », 22 giugno 1929, Tomo XXV, 25, p. 965.

Première contribution au problème de la commutation (Suite et fin). « Revue Générale de l'Electricité », 29 giugno 1929, Tomo XXV, 26, p. 1003.

Ciclo di teoremi sulle potenze complesse. « L'Elettrotecnica », 25 luglio 1929, XVI, 21, p. 491.

Il motore a gabbia folle e l'allacciatore. « L'Elettrotecnica », 5 novembre 1929, XVI, 31, p. 711.

Accouplement magnétique sans frottement. « Bulletin de la Société Française des Electriciens », dicembre 1929, 100.

Les métadynes. Prix 1929 de la Fondation George Montefiore. Estratto dalla « Revue générale de l'Electricité » dell'8 e 15 marzo 1930, Tomo XXVII, p. 355 e p. 395 del 16 e 23 agosto, 22 e 29 novembre e 6 dicembre 1930, Tomo XXVIII, p. 227, p. 260, p. 813, p. 551, e p. 900.

Esquisse sur la Métadyne. « Bulletin de l'Association des Ingénieurs Electricieus sortis de l'Institut Electrotechnique Montefiore », Liège, aprile 1931, 4. Résumé du mémoire couronné par la Fondation George Montefiore à la Session de 1927-1929 et communiqué par l'auteur à la séance de l'A.I.M. du 24 mai 1930.

Corso di Costruzioni Elettromeccaniche. Levrotto e Bella, Torino, Anno Accademico 1936-37, 2 volumi.

Informazione sulla metadinamo. « L'Elettrotecnica », 10 agosto 1937, XXIV, 15, p. 459.

Autarchia e trazione ferroviaria. « Atti della XXVII riunione della S.I.P.S. », Bologna, 4-11 settembre 1938.

Le metadinamo. « Memorie della Reale Accademia delle Scienze di Torino », Serie 2e, Tomo 69, Parte I, 1938-39.

Contributo allo studio dei sistemi elettrici polifasi per mezzo dell'analisi in sistemi simmetrici. « Atti Reale Accademia delle Scienze di Torino », luglio-ottobre 1939, LXXIV, 1, 5, p. 619.

Il collettore di Pacinotti nell'Elettrotecnica di oggi. Relazioni della XXVIII Riunione SIPS, vol. I, Pisa, 11-15 ottobre 1939.

Contributo allo studio della commutazione nelle macchine elettriche. « Atti della Reale Accademia delle Scienze di Torino », LXXV, 1939-40.

Quando e come conviene ricorrere alla metadinamo. « L'Energia Elettrica », gennaio 1940, XVII, p. 26.

Note sul probabile sviluppo della Trazione elettrica. « Rendiconto della Riunione Annuale dell'A.E.I. a Bologna », 1940, XVIII, A2, 118, p. 104.

Nota su certe questioni di sicurezza nei grossi impianti a corrente continua per la metallurgia. « Rendiconti della Riunione dell'A.E.I. a Venezia », 1941, XIX, 3, 133, p. 234.

Elettromeccanica. Edizioni Italiane, Roma. Anno Accademico 1943-44.

Metadyne. Statics. The technology press of M.I.T. and John Wiley and Sons, Inc. New York, 1952.

Some considerations on cybernesis and some examples of applications. « Note, recensioni e notizie dell'Istituto Superiore Poste e Telecomunicazioni », Supplem. n. 4, luglio-agosto 1957.

METADYNE DYNAMICS : LINEAR TRANSIENTS

J. M. PESTARINI

INTRODUCTION

The name of Giuseppe Massimo Pestarini is intimately associated with the invention of the metadyne. I suppose the reader is well acquainted with the design and features of this machine, and I limit myself to quoting the words with which André Blondel, on the occasion of the awarding of the Montefiori Foundation prize to G. M. Pestarini in 1929, commented on the paper on the metadyne: « The paper constitutes a new and important chapter to the theory of the electric machines of direct current, and does honour to its author ».

Towards the end of his hardworking life, G. M. Pestarini began a work of great magnitude: re-ordering his studies on the metadyne.

By 1952, he had published, through the Massachusetts Institute of Technology, the « Metadyne Statics » (J. Wiley & S. Inc., New York), where he exposed in an orderly manner the basic notions concerning the general principles and the static behaviour of certain types of metadynes. Later, he had in mind to develop, in a vaster program and in the same orderly manner, the results of the studies on the « Metadyne Dynamics » and the « Metadyne Combinatory ». Accordingly, a complete series of volumes should have come into existence, dealing with the metadyne in the various possible cases of operation and in its various types, in such a way as to demonstrate the great possibilities and the versatility of these machines.

His premature departure has prevented such complete program from being crealized, thus depriving technical literature of one of the most suggestive works, both for originality and magnitude of conception.

It has been possible, however, to dig up among his papers a part of the original manuscripts, reporting the material that was to form the subject of the successive « series ». Unfortunately, there exist serious gaps in the discovered material, as will become evident from the schematic account given below.

From the examination of the unpublished manuscripts, and particularly of the preface to « Metadyne Dynamics » it has been possible to reconstruct the vast work program that Pestarini had planned to accomplish-the entire « series » was to be subdivided into three main parts, and made up of 7 volumes:

A) Metadyne Statics (Vol. I).

B) Metadyne Dynamics subdivided into:

- Periodic Dynamics (Vol. II).
- Linear Transients Dynamics (Vol. III).
- Non-linear Transients Dynamics (Vol. IV).

C) Metadyne Combinatory subdivided into:

- Systems operating with quasi-periodic law (Vol. V).
- Servomechanisms (Vol. VI).
- Large network stability (Vol. VII).

As stated above, only Vol. I (Metadyne Statics), published in English in 1952, has seen the light. The other inedited volumes, either incomplete or non-existent, are:

- Vol. II (Periodic Dynamics): presumably consisting of five chapters, has only three complete, namely chapters I, II and III.
- Vol. III (Linear Transient Dynamics): presumably consisting of 23 chapters, has only the following chapters complete: I, II, III, IV, X, X*bis*, XI, XII, XV and XXII.
- Vol. IV (Non-linear Transient Dynamics): appears to be non-existent.
- Vol. V, VI, VII (Metadyne Combinatory): appear to be non-existent.

Faced with this situation, we felt duty bound not to neglect collecting the fragments of the this momentous and harmonic construction, which unfortunately no other Mind can attempt to complete. Since the less fragmentary part turned out to be the the so-called Vol. III, which should have dealt with the linear transient Dynamics, it seemed convenient to start with the revision and publication of the manuscripts, beginning with chapters I, II, III and IV of this volume, which in their total make up a more-or-less complete group introducing the discussion of Metadyne Dynamics. The next step in our program is to revise and publish at a future date the other existing chapters in manuscript, both of Vol. III and Vol. II.

To offer a picture, the most possibly complete with the material at our disposal today, of the arguments that were to be the subject of the « Series » of seven volumes, we think it useful to make a list of the titles of the various chapters, including those already published concerning Metadyne Statics.

A) Metadyne Statics (Vol. I).

A/I – *General Considerations valid for all Metadynes.*

- I – Introduction.
- II – Canonical currents.
- III – Electromotive forces in an isotropic metadyne.
- IV – Main characteristics of an isotropic metadyne: a case of rigorously constant rotational speed.
- V – Main Characteristics of an isotropic metadyne: a case of an approximately constant rotation speed, due to a regulation device.
- VI – Metadyne with substantially variable speed.
- VII – Metadynes complete with external connections.
- VIII – Quasi static properties and static stability.
- IX – Considerations on anisotropic metadyne and iron saturation.
- X – On commutation.

A/II – *Special Cases of Metadynes.*

- XI – Introduction.
- XII – Cross Transformer Metadyne, with sustantially constant speed.
- XIII – Some transformer metadynes of the order $m = 4$.
- XIV – Some generator metadynes of the order $m = 4$.
- XV – Some motor metadynes of the order $m = 4$.
- XVI – Some applications using the previously-described metadynes.

XVII – Special metadynes of the order $m = 4$.
XVIII – Some applications of metadynes already described.
XIX – Hyperstatic metadyne: type-8 transformer metadyne.
XX – Some metadynes of a different order than 4 ($m \neq 4$).
XXI – « Rosacea » transformer metadyne.
XXII – The pliodynes.

B) Metadyne Dynamics (Vol. II, III. IV).

B/I – *Periodic Dynamics* (Vol. II).

Preface.

I – Seven fundamental theorems and a lemma on rotating machines fitted with commutator, with special attention to metadynes.
II – Revision of fundamental theorems developed in metadyne statics.
III – Metadyne definition perfected. Fundamental and complementary equations. The theorems of « practical duration of transient ».
IV – (No title or text, only figures exist).
V – (No title or text, only figures exist).

B/II – *Linear Transient Dynamics* (Vol. III).

Preface.

I – « S » Generator Metadyne.
II – Profile of some analytic methods applied in this volume.
III – Generator, motor and transformer metadynes.
IV – Amplifier metadynes (text incomplete).
V, VI, VII, VIII – (No titles, text, or figures exist).
IX – Applications of the magnetizing moment theory (no text or figures).
X – General form of dynamic equations and some of their properties.
X *bis* – Stability criteria.
XI – Applications of Laplace transform.
XII – Applications of the theorem of Laplace transform complex inversion.
XIII – Determination of the roots of a polynomial (no text and figures).
XIV – Applications on the amplifier metadynes (text incomplete).
XV – Some metadynes with simple interconnections.
XVI – Hyperstatic metadyne (text incomplete).
XVII, XVIII, XIX – (No titles, test or figures exist).
XX – Dynamic equations coefficients admitted by Laurent developments (no text or figures exist).
XXI – Dynamic equations coefficients as functions of unknown variables (no text or fiigures exist).
XXII – Some application examples.

B/III – *Non Linear Transient Dynamics* (Vol. IV). (Missing completely).

C) Metadyne Combinatory (Vol. V, VI, VII).

C/1 – *Systems operating with quasi periodic laws* (Vol. V). (Missing completely).
C/2 – *Servomechanisms* (Vol. VI). (Missing completely).
C/3 – *Large network stability* (Vol. VII). (Missing completely).

Even if the publication of the « Metadyne Dynamics) should necessarily refer to some isolated chapters of Pestarini's great but incomplete work, I believe it will equally be useful in showing the clear method followed by him when dealing with problems of some complexity. Such method should not fail to be of advantage in studying electrical machines and plants in general.

In setting about this task of revising the work, efficiently assisted by Eng. Giancarlo Mariotti, I should like to express my most cordial thanks to my Colleague, Prof. Eng. A. M. Angelini, President of A.E.I., who has proposed to me the work of revision and publication, thus kindly giving me the chance to honour the memory of my great Master, the late G. M. Pestarini.

Alfredo Vallini

July 1960.

PROLEGOMENA

Metadynes are described in a group of interrelated volumes; yet each volume is practically autonomous and a few sections reminding definitions, frequently used devices and some theorems, assumed known in this volume, are convenient for a part of readers; the purpose of these prolegomena becomes hence apparent. They comprise following sections:

1.) Subdivision of the investigation.
2.) Fundamental definitions.
3.) Form of the dynamic equations.
4.) Reactivity. Fundamental and complemental equations.
5.) Arbitrary conditions under which an operation occurs; incidents and accidents.
6.) Asymptotic operation.
7.) Synopsis of the previous two volumes.

1.) Subdivision of the investigation.

Any system operating under exchange of energy may be considered as a complex organism whose approximate analysis can be undertaken only through abstraction that considers only a small part of the operating factors and ignores all others.

It is convenient to separate the operating factors into groups, say, G1, G2, G3, the factors of each group having some common characteristics and obeying known laws and to investigate the operation of each group of factors; then to investigate the simultaneous action of many groups. The success of the analysis depends chiefly on the choice of these groups.

This applies to rotating electrical machines, an assembly of elements greatly influenced by mechanical, magnetic, electric, thermic factors to mention the most important.

We are thus led to subdivide the investigation into three main parts, « Statics », where the electric variables are independent of time, « Dynamics », where these variables are functions of time and « Combinatory », where single machines are combined into systems.e Dynamics comprises three subdivisions. In a first one « Periodics », periodic and particularly alternating currents and voltages are considered; in a second one, « Linear Transients » where transient operations are considered which satisfy linear differential equations with constant coefficients; in a third one, « Non Linear Transients, are considered operations satisfying more complicated differential equations.

« Combinatory » comprises many subdivisions depending on the field of application considered, such as: quasi periodic operating systems, servomechanisms, large network stability.

In this volume, « Linear Transients », we will limit our study to some of the electric factors neglecting insulation, dielectric and iron losses and assuming « lumped » inductances and elastances.

2.) Fundamental definitions.

Here are repated some definitions established in the previous volumes:

— We say that a rotating electric machine which shows to an observer, moving along the complete peripheres of the rotor, repetitions of the arrangement of the magnetic and electric elements of the machine, has n cycles denoted by the symbol &.

— Metadyne is a rotating electric machine comprising at least a rotating member, called armature, which, bears at least an armature winding associated with at least a commutator bearing at least three brushes per cycle.

— A metadyne is of order m if it has m brushes per cycle ($m \geqq 3$) and the brushes are indicated by small latin letters $a, b, c, \ldots, m$; while the terminals connected to those brushes are indicated by the corresponding capital latin letters.

— — Canonical current is a current entering to a network by a node, say g, and leaving it in its entirity, by another node, say h; it is indicated by I_{gh}; thus a canonical current entering the metadyne rotor through brush g and leaving it, in its entirity, through brush h, is indicated by the same symbol.

— An electric machine which shows the same magnetic permeance in any radial direction is here called «isotropic». We shall call also isotropic a metadyne which has the same magnetic permeance in any electrical diametrical direction corresponding to an axis of commutation. In the first volume are indicated the necessary and sufficient conditions to be fulfilled for the usual metadynes for operating as if they were isotropic. We shall consider here only isotropic metadynes, except if specifically otherwise indicated.

— There are metadynes whose rotating member is rotating freely in the air, but more often the metadyne comprises two magnetic members roto table with respect to one another, and then for convenience of language we will assume that the armature winding wearing member is rotating, calling it rotor, and that the other magnetic member is fixed, calling it stator. Usually the stator wears windings call stator windings and subdivised into:

— series windings, indicated by S_{oc}^{bd}, where the index bd indicates the canonical current I_{bd} that traverses it, and oc indicates the direction of the axis of the flux it creates when I_{bd} is positive;

— shunt windings, indicated by H_{oc}^{gh}, where the index gh shows the brushes between which said winding is connected, and oc the direction of the axis of the flux it creates when brush g has a higher potential than brush h;

— variator windings, indicated by W_{oc}^{γ}, where the upper index indicates the stator winding current I_{γ}, that traverses it, as generally stator winding currents are discriminated by a Greek index; the lower index has the same significance as above indicated; a variator winding is assumed energized by an external source;

— vinculator windings, indicated by V_{oc}^{γ} with same significance of upper and lower indexes and where the winding is assumed free from connection to any metadyne brush and to any external source.

3.) Form of the dynamic equations.

In considering first the most general case and consecutively coming down to special cases, the development is more concise but it requires a more sustained attention and an already acquired familiarity with the subject permitting to separate in advance the essential from the supplemental.

Investigating first special simple cases then more complicated ones and finally the most general by admitting all possibilities, the development is longer, less elegant but easier to follow by students.

We choose an intermediate proceeding for the lectures and hence for this volume.

The general form of differential equations relative to a metadyne includes terms of the form: $\dfrac{d\,(L\,I)}{dt}$ where $L\,I$ is a flux. Yet there are metadynes having all their m brushes fixed with respect one another and then the inductance coefficient L is constant; but there are matadynes with movable brushes with respect to one another and then L is a function of time. In this part, we will consider only the former case.

4.) Reactivity. Fundamental and complemental equations.

An analysis of the operation may be carried out even when the metadyne is not connected either to external sources nor to external consumers and, contrarily to what happens to most of the conventional electric rotating machines, the analysis may, in the most frequent cases, determine the brush currents when the brush voltages are given or reversely, or more generally, given some brush voltages and some brush currents determine all other brush voltages and brush currents; we say then that the metadyne is a «reactive» one.

It follows hence that most of the conventional electric machines, are non reactive or «areactive». This fact becomes readily apparent when a metadyne is decomposed into a pliodyne as indicated in the first volume.

The group of equations relating brush voltages and brush currents is called the fundamental group and consitutes the nucleus of the metadyne equations. The group of equations relating the brush voltages and the rotor and stator currents of the metadyne to voltages, currents and other characteristic elements of the sources and consumers connected to the metadyne, called complemental group, joint to the fundamental group result in the global group of metadyne equations determining all the characteristics of the operation. We will here limit the development to the simplest relations of the metadyne with external apparatus; in metadyne «Combinatory» more elaborate relation will be considered.

5.) Arbitrary conditions under which an operation occurs; incident and accidents.

Most frequently rotating machines are supplied either by an electrical network or they are driven by a mechanical shaft. In the first case the electrical network is usually characterized by the constancy of the value of an electric characteristic quality as by constant voltage, or by constant current or by constant power; in the second case the mechanical energy delivered by the shaft is characterized usually by the constancy of speed or the constancy of the torque, or the constancy of the power or by some other characteristic. Accordingly special incidents and accidents are involved giving rise to a transient operation. Thus frequent accidents for constant voltage networks supplying many consumers in parallel with one another, are the abrupt imperition of the voltage, and its abrupt dimination; possible accidents are short-circuits and overvoltages. Similarly a constant speed prime mover may accidentally stop or run at an unsafe high speed; or a constant torque prime mover may show abrupt or periodical changes of the torque.

The investigation of the corresponding transient operation, main object of this volume, reveals the stress impressed on the machine under examination and informs about the final behaviour of the machine, thus it informs whether, after the incident or the accident, the operation returns to its previous form or not, and accordingly we use to say that the operation is stable or unstable.

This simple definition of the stability is satisfactory in most practical cases but it is incomplete. When stability, as the main object will be considered, later on, a wider determination will be given and accordingly a more exhaustive analysis will be developed.

Instead of the simple condition of the constancy of a characteristic electrical quantity during the operation here above mentioned, there are other more complicated conditions, and we may here mention that the advent of metadynes brought forward relations between supplied current and impressed voltage graphically representable by a conic or a more elaborated curve, relations particularly adapted for special applications. Such applications will be closer considered in « Metadyne Combinatory ».

6.) Asymptotic operation.

A « static » operation is the asymptotic form of a dynamic operation whenever the abstraction of the infinite preexistence is avoided.

A careful investigation of the metadyne static equations may lead us to recognise the probable nature of the dynamic operation. Thus if static equations lead to definite currents or voltages we may conclude that there is a probable building up; if they lead to indefinite values we may conclude that oscillations are probable.

Let us take the series dynamo connected to an external resistance as an example. The static equation is:

$$n K I - R I = 0$$

which for $I \neq 0$ yelds necessarily $n K = R$, for which case I is indetermined. In this case I may be infinite or may oscillate; but in order to have oscillations we must have the possibility of storing energy in at least two different locations. If the speed, n, is kept rigidly constant there is no possibility of mechanical storage of energy and as a resistor dissipates and does not store energy, there must be necessarily a building up of current I towards infinity if $n > R/K$. If the prime mover to which the series dynamo is coupled is of a limited nominal power of the same order as the series dynamo itself, then oscillations are very probable, the more probable the larger the self inductance of the dynamo circuit.

Thus static equations may yield indications on the stability of operation. Much more does the investigation of the transient operation. Thus although stability will be considered more closely in a further volume, yet it will be here frequently mentioned whenever the case lends itself to a simple discussion.

Considering the general case we may say that when time t tends to infinite either there is an asymptotic operation to which the operation at a finite time approaches continuously when time increases, or there is a transient operation of infinite duration. In fact in case there is not asymptotic operation, in other words if there is no law to which the operation tends to comply when time increases indefinitely, then some cause, arbitrarily changing, must intervene in order to change the operation with exclusion of any law, and therefore we have a transient operation at any value of time without limit of magnitudes.

The asymptotic operation may have three possible forms:

in a first form the variables tend to a finite constant value;

in a second form the variables tend to an infinite value;

in a third form the variables take values continuously changing and oscillating between two limits when time increases indefinitely, hence the asymptotic operation is a periodic one. The limits of the values in the periodic law may be finite of infinite.

Accordingly we may say that we have a finite direct current, an infinite direct current and a periodic current operation, respectively. In the latter case we may distinguish finite from infinite limits.

7.) Synopsis of the two previous volumes.

For the convenience a brief descriptive comment of the two previous volumes is here given.

In « *Metadyne Statics* » direct current operation is considered. The concept of « canonical currents » is frequently used. A set of canonical currents may replace the existing set of non canonical currents and simplify the analytical investigation.

While the basic normal operation of isotropic (or equivalent to isotropic) dynamos, alternators, synchronous generators and asynchronous motors may generally be analytically represented by a single equation, the operation of metadynes provided with m brushes per cycle, i.e. of order m, requires generally m-1 « fundamental » equations, defining a « global characteristic », which referred to an hyperspace gives a graphic representation to the existing relations between the $2m - 2$, generally variables $x_1, x_2, \ldots x_{2m-2}$, brush currents and brush voltages. In case there are no stator windings the sections of the global characteristic parallel to two axes, corresponding to two of said variables, say x_g and x_h, are straight lines passing through the origin, in case there are stator windings said sections are straight lines which may not pass through the origin.

In case there is a stator « speed regulator » winding traversed by a current highly sensitive to small speed variations, the above indicated sections are conics, a valuable property that widens the field of possible applications.

Commutation in a metadyne may happen at any point of the magnetic poles and not necessarily at the interpole space; thus commutation appears practically impossible. A special chapter deals with this problem, establishes some theorems and describes a complete solution permitting metadynes to commutate as well, at least, as dynamos and some times better.

Many particular cases of metadynes are described and applications mentioned where the machine operates as a generator with various kinds of current to voltage relations, or a motor with a variety of torque speed characteristics, or as a mere transformer, transforming electric power into electric power, or as a complex transformer transforming simultanuously mechanical into electrical power, and electrical power into electrical power of different characteristics.

A special category of metadynes is considered, the amplifier metadynes, and their most elemental static properties are analysed.

Another special category is examined characterised by the fact that the fundamental and complemental equations do not suffice for defining brush currents and voltages; such metadynes are referred to as « hyperstatic metadynes ». In this volume further attention is given to this category.

A system comprising a group of dynamos each of which has a field winding traversed by the armature current of another dynamo of the same group is said to constitute a pliodyne. A metadyne is equivalent to a pliodyne and reversely, when the equations ruling their permanent operation are the same. In the last chapter of « Metadyne Statics » is shown how to a given metadyne there are generally more than one equivalent pliodyne. Equivalent pliodynes are always heavier than the corresponding metadyne and more cumbersome.

In « *Metadyne Periodics* », the operation under alternating current is considered. The form of the corresponding equation is generally very different when all brushes are fixed with respect to one another and when they are movable. In the former case equations are simple and may be readily treated by the familiar method using a Fresnel diagram and Steinmetz's complex number equations. An investigation of the operation of these metadynes shows that the characteristics obtained approach the ones found in « Metadyne Statics » whenever the frequency of the alternating current reduces towards zero; that there is almost always a creation of reactive power which may be used for obtaining an arbitrarily determined power factor; that there are singular values of speed and frequency for which the modules of currents increases to a peak value, and we then say that there is an « electromechanical resonance ».

« Virtual powers », i.e. powers obtained by transforming according to predetermined laws, voltages and currents, are investigated and some theorems established demonstrating the invariancy of some virtual powers within determined domains.

The formula deduced from these theorems permit to readily solve problems which would be laborious to solve by the known methods. Such problems arise particularly with metadynes having brushes movable with respect to one another. The operation of the last category of metadynes is again analysed by another method using multiple Fresnel diagrams and groups of Steinmetz's complex number equations.

To the new machines brought forward in « Metadyne Statics, » may others are added in « Metadyne Periodics ».

The fundamental theorems and rules developed in the first volume are reconsidered and widened in the second and again used and completed in this third volume.

JOSEPH MAXIMUS PESTARINI

PART ONE

TRANSIENT OPERATION OF SIMPLE METADYNE UNITS. (1)

CHAPTER I

THE « S » GENERATOR METADYNE

The investigation of the characteristics of the dynamic behaviour of each component element of a metadyne is the main purpose of this chapter. A simple case is chosen, the « *S* » generator metadyne, and the action of the armature windings, and that of the various stator windings are separately examined and commented. No doubt the simultaneous operation of many windings gives rise to new characteristics due to their interaction, but the basic characteristics pointed out in their exclusive operation, generally remain predominant and give an imperfect, yet easy, preview of the complex operation of the complete system.

Some new factors of operation and their symbols are introduced such as the « reactivity resistance » and the kinetic resistances of the metadyne because they facilitate the development, unify the form of many formulae, and point out important characteristics of a metadyne. The analysis requires only brief equations easy to memorize and to transfuse into simple physical visualization of the phenomenon; the sections of this chapter cease as soon as the analysis becomes too complex.

1.) *S* generator metadyne driven at constant speed; asymptotic behavior.
2.) *S* generator metadyne driven at constant speed; transient behavior.
3.) Two dynamos in cascade.
4.) Addition of series stator windings in the direction of the armature flux.
5.) Addition of series stator windings having their magnetic axis electrically normal to the armature flux axis.
6.) Metadyne's reactivity as a stability factor.
7.) Behavior of a current traversing a coil supplied by a reactive metadyne.
8.) Action of vinculators comprising resistance and inductance.
9.) Permanent oscillations.
10.) Shunt windings.
11.) Insertion of capacitors into the circuit.
12.) Diagonal vinculator.

1.) *S* GENERATOR METADYNE DRIVEN AT CONSTANT SPEED; ASYMPTOTIC BEHAVIOR.

Let us first consider the simplest case where a secondary variator winding is the only stator winding; it is energized by some external source and its mutual induction with the armature winding is set to zero by means of a transformer, T, as shown by the diagram of fig. 1; the consumer is a reactor. Let R_p, L_p, R_s, L_s be the characteristics of the primary and secondary circuits respectively. The static equations are:

$$(1)\qquad \begin{aligned} n\, K^{\gamma}_{ac}\, I_{\gamma} &= n\, K^{bd}_{ac}\, I_{bd} + R_p\, I_{ac} \\ 0 &= n\, K^{ac}_{bd}\, I_{ac} - R_s\, I_{bd} \end{aligned}$$

where n denotes the speed in revolutions per second and the K' s are the coefficients corresponding to the windings of the machines. These coefficients are considered in the previous volumes and their upper index indicates the corresponding current that induces the e.m.f. while the lower index indicates the brushes between which said e.m.f. is induced. System (1) yields:

$$(2)\qquad \frac{I_{ac}}{I_{\gamma}} = \frac{n\, K^{\gamma}_{ac}\, R_s}{n^2 K^2 + R_s R_p} \qquad \frac{I_{bd}}{I_{\gamma}} = \frac{n^2 K\, K^{\gamma}_{ac}}{n^2 K^2 + R_s R_p}$$

where $K = K^{ac}_{bd} = - K^{bd}_{ac}$, an essentially positive coefficient of the armature winding and the positive direction of rotation are dextrorse.

Values (2) are always finite as the determinant of the coefficients of equations (1) is different from zero:

$$(3)\qquad \begin{vmatrix} R_p & n\,K \\ n\,K & -R_s \end{vmatrix} \neq 0.$$

We may deduct that the operation is quite probably always stable, under the conditions here assumed.

(1) The general title concern the four chapters, related on the present monography and probably some other missin chapters; in another section of the III volume would be apart interested the problem of many metadynes interconnected each other.

2.) *S* GENERATOR METADYNE AT CONSTANT SPEED; TRANSIENT BEHAVIOR.

Considering again the scheme of fig. 1, let us assume that there is a switch in the primary circuit abruptly closed, as we will find indeed, that in most practical cases a switch is inserted in the primary circuit.

The corresponding transient satisfies following equations starting from $t = 0$:

$$(4)\qquad \begin{aligned} & n K_{ac}^{\gamma} I_{\gamma}(t) - n K I_{bd}(t) - \\ & \quad - \left(R_p + L_p \frac{d}{dt}\right) I_{ac}(t) = 0 \\ & n K I_{ac}(t) - \left(R_s + L_s \frac{d}{dt}\right) I_{bd}(t) = 0 \end{aligned}$$

where $I(t)$ indicates a current function of time; let us assume, further, that:

$$I_{ac}(t) = I_{bd}(t) = 0 \quad (\text{for } t < 0)$$

and that I_{γ} was beforehand brought to its chosen value. The abrupt closing of the switch inserted into the primary is then equivalent to the assumption that the current of the secondary variator winding was zero for

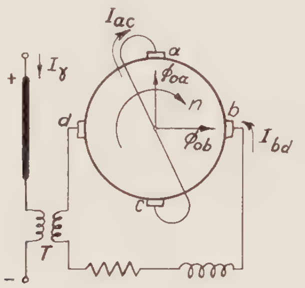


Fig. 1.

$t < 0$ and that it becomes abruptly equal to I_{γ} for $t = 0$ and remains such hereinafter; in other words the closing of the primary swich is equivalent to the assumption that the function $I_{\gamma}(t)$ is a Heavisidean function whose value is constant, I_{γ}, for $t > 0$. We are also assuming that the flux due to a current magnetising a perfectly laminated stack, is synchronous to the current. In other words we neglect, for the time being, the Foucault currents into the laminations. We call here Heavisidean functions the functions which are nil for any value of time earlier than an arbitrarily predetermined instant, t, at which instant they reach abruptly any other predetermined value and are arbitrarily determined hereinafter; the instant t is called the critical value of time for the Heavisidean function considered. Such functions are here indicated by adding after the parenthesis, a lower index with the value of the critical instant. Thus all previous assumptions may be written as follows:

$$(5)\qquad I_{ac}(t)_0; \quad I_{bd}(t)_0; \quad I_{\gamma}(t)_0$$

where the two former functions are the unknown and the latter is known to satisfy:

$$(6)\qquad I_{\gamma}(t)_0 = I_{\gamma} \text{ for } t > 0$$

I_{γ} being a constant.

Under these conditions, the Laplace transforms of equations (4) are:

$$(7)\qquad \begin{aligned} & (R_p + \tau L_p) I_{ac}(\tau) + n K I_{bc}(\tau) = \\ & \qquad = n K_{ac}^{\gamma} I_{\gamma} \frac{1}{\tau} \\ & n K I_{ac}(\tau) - (R_s + \tau L_s) I_{bd}(\tau) = 0 \end{aligned}$$

where the symbol $I(\tau)$ indicates the Laplace transform of the function of time $I(t)$, the greek letter τ being a complex.

Solving equations (7) we obtain:

$$(8)\qquad \begin{aligned} \frac{I_{ac}(\tau)}{I_{\gamma}} &= \frac{1}{\tau} \cdot \frac{n K_{ac}^{\gamma} (R_s + \tau L_s)}{n^2 K^2 + (R_p + \tau L_p)(R_s + \tau L_s)} \\ \frac{I_{bd}(\tau)}{I_{\gamma}} &= \frac{1}{\tau} \cdot \frac{n^2 K K_{ac}^{\gamma}}{n^2 K^2 + (R_p + \tau L_p)(R_s + \tau L_s)} \end{aligned}$$

The three roots of the denominator of the last fractions are:

$$(9)\qquad \begin{aligned} & \tau_0 = 0; \\ & \tau_1 \text{ and } \tau_2 = \frac{1}{2}\left[-\left(\frac{1}{T_p} + \frac{1}{T_s}\right) \pm \right. \\ & \left. \pm \sqrt{\left(\frac{1}{T_p} + \frac{1}{T_s}\right)^2 - \frac{4 n^2 K^2}{L_p L_s}}\,\right] \end{aligned}$$

where $T_p = \dfrac{L_p}{R_p}$ and $T_s = \dfrac{L_s}{R_s}$ are the time constants of the primary and the secondary circuits respectively.

The last term under the radical of equation (9) may be put under following form:

$$10)\qquad \frac{4 n^2 K^2}{L_p L_s} = \frac{4 R_r^2}{L_p L_s} = 4 \frac{1}{T_r^2}$$

where the resistance R_r is due to the reactivity of the metadyne. Its action will further be discussed; will call R_r and T_r reactivity resistance of the metadyne and correspondingly reactivity time constant of the metadyne combined with the external circuit.

In case the quantity under the radical figuring in (9) is positive, i.e., if $4 R_r^2 < I_p L_s \left(\frac{1}{T_p} - \frac{1}{T_r}\right)^2$ the inverse Lalace transform yields the solution:

$$(11)\qquad \begin{aligned} \frac{I_{ac}(t)_0}{I_{\gamma}} &= A_1 + B_1 e^{\tau_1 t} + C_1 e^{\tau_2 t} \\ \frac{I_{bd}(t)_0}{I_{\gamma}} &= A_2 + B_2 e^{\tau_1 t} + C_2 e^{\tau_2 t}. \end{aligned}$$

In case $4 R_r^2 > L_p L_s \left(\frac{1}{T_p} - \frac{1}{T_s}\right)^2$, the radical is negative, say equal to $-2\,\omega^2$, the solution may take following form:

$$(12)\qquad \begin{aligned} \frac{I_{ac}(t)_0}{I_{\gamma}} &= A_1 + B_3 e^{\tau_3 t} \sin(\omega t - \varphi_1) \\ \frac{I_{bd}(t)_0}{I_{\gamma}} &= A_2 + B_4 e^{\tau_3 t} \sin(\omega t - \varphi_2) \end{aligned}$$

where:

$$\tau_3 = -\frac{1}{2}\left(\frac{1}{T_p} + \frac{1}{T_s}\right) = -\frac{1}{T_q}.$$

$$\omega = \frac{1}{\sqrt{2}} \sqrt{\frac{4\,n^2 K^2}{L_p L_s} - \left(\frac{1}{T_p} - \frac{1}{T_s}\right)^2}$$

and $$T_q = 2\,\frac{T_p T_s}{T_p + T_s}.$$

Finally if the radical is zero, the solution becomes:

$$(13)\qquad \begin{aligned} \frac{I_{ac}(t)_0}{I_\gamma} &= A_1 + B_5\, e^{-\frac{t}{T_q}} + C_5\, t\, e^{-\frac{t}{T_q}} \\ \frac{I_{bd}(t)_0}{I_\gamma} &= A_2 + B_6\, e^{-\frac{t}{T_q}} + C_6\, t\, e^{-\frac{t}{T_q}} \end{aligned}$$

wehre the capital letters A, B and C are integration constants. In any case the transient terms tend to zero and we may conclude that the operation is always stable under the assumed conditions.

3.) Two dynamos in cascade.

It is convenient for our comments to consider two dynamos in cascade each of which has the same armature as the metadyne, same air gap, same external circuit and rotates at same speed, n. fig. 2 show the scheme. As there is practically same reluctance of the magnetic circuit we may assume that the inductor of the first dynamo bears a coil similar to the secondary variator winding of the metadyne and traversed by a current I_δ. Let us call I_ε the current supplied by the armature

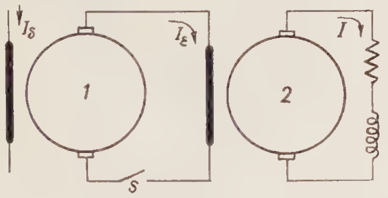


Fig. 2.

of the first dynamo and I the current supplied by the armature of the second dynamo to the external circuit. Static equations are:

$$(14)\qquad n\,K^\delta I_\delta = R_\varepsilon I_\varepsilon\,; \qquad n\,K^\varepsilon I_\varepsilon = R\,I$$

with obvious meaning of symbols n, K^δ, R_δ, K^ε, R and following asymptotic solution:

$$(15)\qquad \frac{I}{I_\delta} = \frac{n^2 K^\varepsilon K^\delta}{R\,R_\varepsilon}.$$

Comparing with formulae (2), we find now a smaller denominator.

Let us now establish the exciting current I_ϵ at its steady value and close abruptly switch S. Repeating the arguments of the previous paragraph we obtain following Laplace transforms for corresponding dynamic equations:

$$(16)\qquad \begin{aligned} \frac{n\,K^\delta I_\delta}{\tau} &= (R_\varepsilon + \tau\, L_\varepsilon)\, I_\varepsilon(\tau) \\ n\,K^\varepsilon I_\varepsilon(\tau) &= (R + \tau\, L)\, I(\tau) \end{aligned}$$

hence:

$$(17)\qquad \frac{I(\tau)}{I_\delta} = \frac{n^2 K^\delta K^\varepsilon}{L\,L_\varepsilon} \cdot \frac{1}{\left(\tau + \frac{R}{L}\right)\left(\tau + \frac{R_\varepsilon}{L_\varepsilon}\right)\tau}.$$

Let us consider the following decomposition into simple fractions:

$$(18)\qquad \frac{1}{(\tau - \alpha)\cdot(\tau - \beta)\cdot\tau} = \frac{A}{\tau - \alpha} + \frac{B}{\tau - \beta} + \frac{C}{\tau}$$

we obtain:

$$(19)\qquad A = \frac{1}{\alpha(\alpha - \beta)}; \quad B = \frac{1}{\beta(\beta - \alpha)}; \quad C = \frac{1}{\alpha\beta}$$

except for the case $\beta = \alpha$ for which following decomposition

$$\frac{1}{(\tau - \alpha)^2\,\tau} = \frac{D}{(\tau - \alpha)^2} + \frac{E}{\tau - \alpha} + \frac{F}{\tau}$$

yields:

$$(20)\qquad D = \frac{1}{\alpha}; \qquad E = -\frac{1}{\alpha^2}; \qquad F = \frac{1}{\alpha^2}.$$

Correspondingly we obtain following two solutions:

In case $\frac{R}{L} = \frac{1}{T} \neq \frac{R_\varepsilon}{L_\varepsilon} = \frac{1}{T_\varepsilon}$ there is:

$$(21)\qquad \frac{I(t)_0}{I_\delta} = \frac{n^2 K^\varepsilon K^\delta}{R\,R_\varepsilon}\left[1 + \frac{1}{\frac{T_\varepsilon}{T} - 1}\, e^{-\frac{t}{T}} + \frac{1}{\frac{T}{T_\varepsilon} - 1}\, e^{-\frac{t}{T_\varepsilon}}\right]$$

If $T = T_\varepsilon$, there is:

$$(22)\qquad \frac{I(t)_0}{I_\delta} = \frac{n^2 K^\varepsilon K^\delta}{R\,R_\varepsilon}\left[1 - e^{-\frac{t}{T}} - \frac{t}{T}\, e^{-\frac{t}{T}}\right].$$

In order to compare the results, let us determine the integration constants of the solutions (11), (12), and (13). We may, for convenience, put:

$$-T_\alpha = \frac{1}{\tau_1}; \qquad -T_\beta = \frac{1}{\tau_2};$$

and write the second of equations (11) as follows:

$$(23)\qquad \frac{I_{bd}(t)_0}{I_\gamma} = \frac{n^2 K\, K^\gamma_{ac}}{n^2 K^2 + R_s R_p} \cdot \left[1 + \frac{1}{\frac{T_\beta}{T_\alpha} - 1}\, e^{-\frac{t}{T_\alpha}} + \frac{1}{\frac{T_\alpha}{T_\beta} - 1}\, e^{-\frac{t}{T_\beta}}\right]$$

The coefficient outside the parentesis may be written as follows:

$$\frac{n^2 K K^{\gamma}_{ac}}{R_p R_s + R_r^2} = \frac{K^{\gamma}_{ac}}{K} \cdot \frac{R_r^2}{R_p R_s + R_r^2} .$$

Form (23) is valid for all cases except when the radical of (9) is zero, for which case the second equation (13) with explicit integral constants becomes:

$$\frac{I_{bd}(t)_0}{I_\gamma} = -\frac{n^2 K K^{\gamma}_{ac}}{R_p R_s} \left[1 - e^{-\frac{t}{T_q}} + \frac{t}{T_q} e^{-\frac{t}{T_q}} \right] \tag{24,1}$$

while the critical reactivity resistance $R_{r,c}$ and time constant, $T_{r,c}$ are defined by following equation:

$$R_{r,c}^2 = \frac{1}{4} \left(\frac{1}{T_p} - \frac{1}{T_s} \right)^2 \cdot L_p L_s \qquad T_{r,c} = T_q . \tag{24,2}$$

Let us limit our comments here to the practically important case where

$$T_s \gg T_p \text{ and similarly } T \gg T_\varepsilon . \tag{25}$$

Under these conditions the second exponential of equation (21) may be neglected with respect to the first one and the latter may be simplified as follows:

$$\frac{- n^2 K^{\delta} K^{\varepsilon}}{R R_\varepsilon} e^{-\frac{t}{T}} \tag{26}$$

The time constants T_α and T_β characterizing the operation of the metadyne depend not only from the time constants T_p and T_s of the primary and the secondary circuit but also on the the reactivity resistance R_r or better on the reactivity time constant T_r; thus we will take the latter as independent variable. This variable shows a critical value, $T_{r,c}$ for which the radica- of (9) vanishes passing from real to imaginary values. This critical value $T^2_{r,c}$ is determined by:

$$\frac{4}{T_{r,c}^2} = \left(\frac{1}{T_p} - \frac{1}{T_s} \right)^2 \tag{27}$$

hence:

$$T_{r,c}^2 = \pm 4 \frac{T_p^2 T_s^2}{(T_p - T_s)^2} . \tag{28}$$

The differential equations (4) from which we started, consider only resistances and inductances for the sake of an easier beginning; in alternating current metadynes capacitors are generally inserted into the circuits and the corresponding differential equations lead to more laboring comments developed later on. If T_p and T_s remain real, the critical reactivity time constant remains real as well.

When T^2_r varies from infinite to $T^2_{r,c}$, T_α and T_β vary from T_p and T_s respectively to following critical values:

$$T_{\alpha,c} = T_{\beta,c} = 2 \frac{T_p T_s}{T_p + T_s} = 2 T_p \left[1 - \frac{T_p}{T_s} + \left(\frac{T_p}{T_s} \right)^2 \cdots \right]. \tag{29}$$

For lower values, until zero, T_α and T_β are complex whose real part remains constant and whose imaginary part increases from zero to $\pm j\infty$. The diagram of fig. 4.a shows the real and the imaginary part of T_α and T_β, respectively:

$$R\, T_\alpha , \quad R\, T_\beta , \quad I\, T_\alpha , \quad I\, T_\beta \tag{30}$$

as functions of T_r^2 for real positive values of L_p, L_s, R_p, R_s and for $T_p = 1/10\ T_s$. When the reactivity resistance R_r increases from zero to infinite, the larger time constant of the two exponentials figuring in solution (23) decreases from T_s to a value smaller than $2\,T_p$, remains constant at this value when R_r is larger than the critical reactivity resistance, $R_{r,c}$, satisfying equation (28), and the transient becomes oscillatory. For the same variation of R_r the smaller time constant of the two exponentials of (23) increases from T_p to less than $2\,T_p$. We may conclude that the operation is then always stable.

When $T_p = T_s$ while R_r increases from zero to infinite, the operation is stable and the transient is always oscillatory, except for $R_r = 0$, for which case the transient takes the from (24,1).

4.) Addition of coniugate series stator windings, in the direction of the armature flux.

Any series stator winding may be decomposed into two others, one having its magnetic axis along the primary commutating axis and another having its magnetic axis along the secondary commutating axis. Therefore the four series windings 2, 3, 4 and 5 of fig. 3,

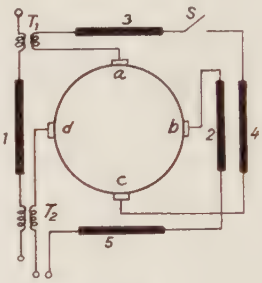


Fig. 3.

may represent any combination of series windings for an S generator metadyne.

Let us first consider the action of the secondary series winding 2 having its magnetic axis along the secondary commutating axis and being traversed by the secondary current, I_{bd}, while all other series windings are supposed non existing.

The direct current asymptotic equations may be written as follows:

$$\begin{aligned} n K^{\gamma}_{ac} I_\gamma &= n K I_{bd} + n K^{\beta}_{ac} I_{bd} + R_p I_{ac} \\ 0 &= n K I_{ac} - R_s I_{bd} . \end{aligned} \tag{31}$$

The coefficient K^{β}_{ac} corresponding to winding 2 may be positive or negative i.e. the winding may be an amplifier or a compensator.

$$D = \begin{vmatrix} R_p & n (K + K^{\beta}_{ac}) \\ n K & - R_s \end{vmatrix} = - R_p R_s - n^2 K (K + K^{\beta}_{ac}). \tag{32}$$

It is always different from zero when winding 2 is an amplifier or an hypocompensator; it becomes zero for

following critical values of an hypercompensator:

$$K^{\beta}_{ac,c} = -K - \frac{R_p R_s}{n^2 K}. \tag{33}$$

For values of K^{β}_{ac} smaller than K, the corresponding reactivity resistance has an imaginary value:

$$R_r = j n \sqrt{K \left| K^{\beta}_{ac} + K \right|}. \tag{34}$$

When (33) is satisfied the value of the solution $\frac{I_{bd}(t)}{I_\gamma}$ tends to infinity leading us to expect a building up. Note that the critical value (33) is practically reached when the compensation is complete (100%) because R_p is very small in almost all constructed metadynes. At the same time the solution of equations (31) deprived of the first member $n K^{\gamma}_{ac} I_\gamma$, takes the form 0/0, i.e. it becomes indefinite hinting to probable simultaneous oscillations because there are at least two different locations for energy storage, the primary and the secondary magnetic circuit which although have a common part, have a large non common part. If the prime mover is further unable to drive at constant speed for any load, other oscillations, of electromechanical character, may coexist.

The corresponding critical values, n_c, of the speed, determined by equation (33) are real but practically very small when winding 2 is an hypocompensator with a degree of compensation fairly lower than 100%, because R_p is generally very low. When the degree of compensation approaches 100%, n_c increases rapidly reaching the field of the values the rotational speed is given in the practice.

The ratios of asymptotic primary and secondary currents to current I_γ are:

$$\begin{aligned} \frac{I_{ac}}{I_\gamma} &= \frac{n K^{\gamma}_{ac} R_s}{n^2 K (K + K^{\beta}_{bd}) + R_p R_s} \\ \frac{I_{bd}}{I_\gamma} &= \frac{n^2 K K^{\gamma}_{ac}}{n^2 K (K + K^{\beta}_{bd}) + R_p R_s} \end{aligned} \tag{34,1}$$

and fig. 5 and 6 represent them for the case of an hypercompensator.

The critical values n_c correspond to an electromechanical resonance.

Assuming now switch S is abruptly closed when I_γ is established we obtain a transient operation whose differential equations have following Laplace transform:

$$\begin{aligned} (R_p + \tau L_p) I_{ac}(\tau) + n (K + K^{\beta}_{ac}) I_{bd}(\tau) &= \\ = n K^{\gamma}_{ac} I_\gamma \frac{1}{\tau} & \\ n K I_{ac}(\tau) - (R_s + \tau L_s) I_{bd}(\tau) &= 0. \end{aligned} \tag{35}$$

The determinant, D, of the coefficients of equations (35) may be written as follows:

$$\begin{aligned} -D = \tau^2 L_p L_s + \tau (R_p L_s + R_s L_p) + & \\ + R_p R_s + n^2 K (K + K^{\beta}_{ac}) = & \end{aligned} \tag{36}$$

The square of reactivity resistance, R_r, and of reactivity time constant T_r, defined by:

$$R_r^2 = n^2 K (K + K^{\beta}_{ac}); \quad T_r^2 = \frac{L_p L_s}{n^2 K (K + K^{\beta}_{ac})}; \tag{37}$$

take negative values when $K^{\beta}_{ac} < -K$.

The corresponding diagram, given by fig. 4.b covers now the whole coordinate plane; let us construct the left semiplane branches; for $R_r = 0$ and $T_r^2 = \infty$ we have $T_\alpha = T_p$ and $T_\beta = T_s$ with form (22). When R_r^2 takes negative values with increasing modulus, the solution keeps form (24,2) and the operation is stable until following critical value is reached:

$$T^2_{r,c} = -T_p T_s \tag{38}$$

for which one of the roots, say root T_α, becomes zero and remains hereinafter positive for increasing modulus of R_r^2, showing a building up of primary and secondary currents towards infinite, building up more violent for larger values of the modulus of R_r^2. The diagram on the right semiplane is the same as that of figure 4.a For a determined external circuit, the diagram of fig. 4.a admits only one variable, the speed n, while the diagram of fig. 4.b admits two variables, the speed n, and the coefficient K^{β}_{ac}.

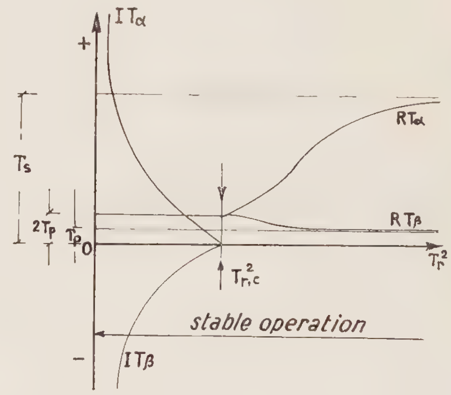


Fig. 4. *a*

It is important to note here that equation (38) coincides with equation (33) extracted from the direct current, permanent state equations.

Note further that the reactivity time constant of constructed metadynes take values equal to a small fraction of a second when the metadyne has neither compensators nor amplifiers.

We may generally conclude that the secondary compensating winding reduces the quick response ability of the metadyne and causes instability almost immediately as the degree of compensation exceeds 100%.

In fact let us modify continuously the value of R_r^2 from 0 to $+\infty$, the representative points starting from the infinity at right, to which corresponds $T_r^2 = +\infty$ out a degree of compensation 100%, traverse the whole semiplane at right reaching the axis of the ordinates when $R_r^2 = +\infty$, $T_r^2 = 0$ and the degree of amplification becomes infinite. All this region I (fig. 7), is a domain of stability. Starting now from the infinity at left, $T_r^2 = -\infty$, i.e. from the value $R_r^2 = 0$ and 100% degree of compensation, and letting R_r^2 take negative values, the representative points traverses the region III, still a domain of stability until a line $p\,q$ is reached. For increasing modules of the negative value of R_r^2, the representative points traverse region II a domain of instability. R_r^2 reaches the critical value:

$$R_r^2 = n^2 K (K + K^{\beta}_{ac}) = -R_p R_s$$

for K^{β}_{ac} slightly smaller than $-K$ because both R_p and R_s are small as compared to $n K$, and particularly R_p is of the order of 1% of $n K$ and even smaller. Thus

instability is reached usually when the degree of compensation exceeds 100% [2] of less than 1%, and it becomes more evident when the degree of compensation increases.

On the contrary an amplifier enchances quick response. where the reactivity time constant may now take real and imaginary values even with compensators compensating the armature more than 100%. The action of the primary compensator can modify radically the action of the secondary one, amplifiers being comprised

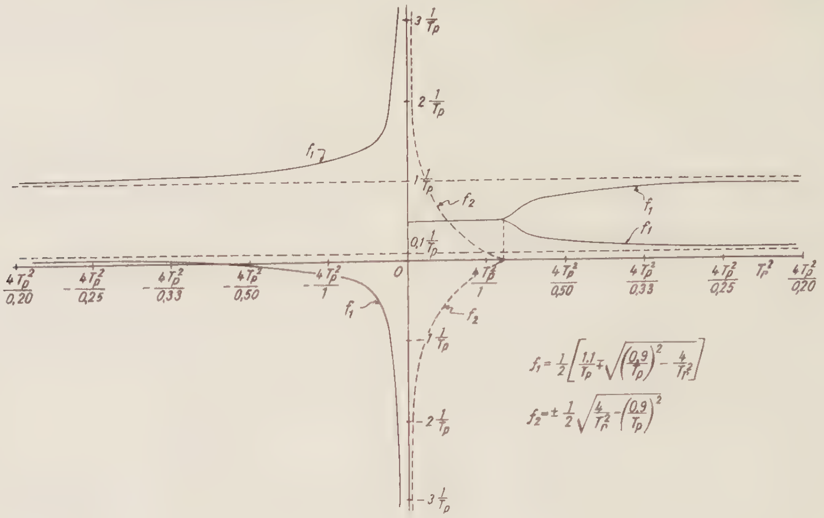


Fig. 4. b

Let us now consider both series winding 2 and 3 of fig. 3. The Laplace transform of the corresponding differential equations, under the same boundary conditions assumed before, are:

$$
\begin{aligned}
&(R_p + \tau L_p)\, I_{ac}(\tau) + n\,(K + K^{\beta}_{ac})\, I_{bd}(\tau) \\
(39)\qquad &= n\, K^{\gamma}_{ac}\, I_\gamma \frac{1}{\tau} \\
&n\,(K + K^{\alpha}_{bd})\, I_{ac}(\tau) + (R_s + \tau L_s)\, I_{bd}(\tau) = 0
\end{aligned}
$$

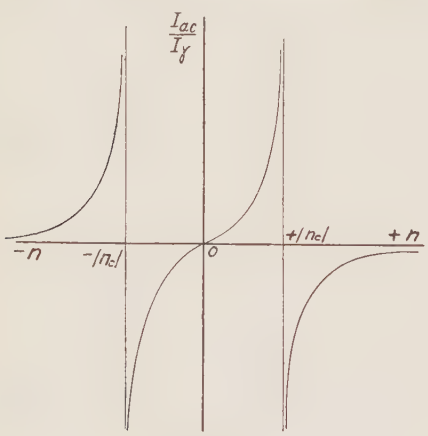


Fig. 5.

and the determinant of the coefficients is:

$$
\begin{aligned}
-D &= n^2 (K + K^{\beta}_{ac})(K + K^{\alpha}_{bd}) + \\
&+ (R_p + \tau L_p)(R_s + \tau L_s) \\
&= L_p L_s \left[\left(\tau + \frac{1}{T_p}\right)\left(\tau + \frac{1}{T_s}\right) + \right. \\
&\left. + \frac{1}{T_p \cdot T_s} + \frac{1}{T_r^2}\right]
\end{aligned}
$$

by considering them as compensators with negative degree of compensation.

The corresponding diagram covers both semiplanes and it is the same as that of fig. 4.b except when there is always $K^{\beta}_{ac} = K^{\alpha}_{bd}$, for which case the diagram covers only the right hand semiplane and it is the same as that of fig. 4.a.

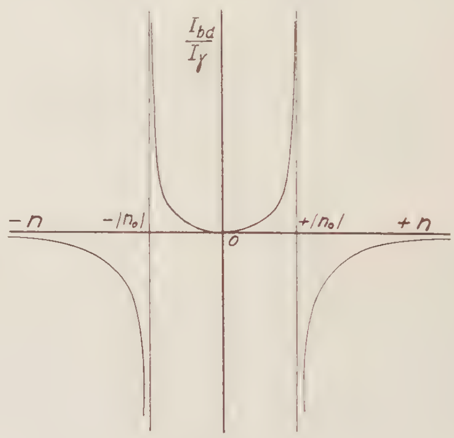


Fig. 6.

All series windings in a metadyne may be « concentrated », i.e. coils interlinked with one or more polar segments complete, or they may be « distributed », i.e. coils distributed along the air gap and thus interlinked with only a part of one or more polar segments. The construction of the compensators, with positive degree of compensation, under the « distributed » form, has the advantages of reducing the iron saturation on the horns of the polar segments and of more equally distributing the voltage induced between consecutive segments of the commutator and thus easier admitting high voltages and reducing flash over possibilities. The construction of « distributed » winding requires higher cost and larger sizes than the manufacture of « concentrated » ones.

It is now obvious that amplifiers, with positive degree amplifications are more convenient under the « concentrated » form.

(2) An operation with a complete 100% compensation, is an operation at the border.

5.) Addition of coniugate and quadrature series stator windings having their magnetic axis electrically normal to the armature flux axis.

Such windings are stabilizers and stimulators.

The Laplace transform of the corresponding differial equations, under the usual boundary conditions, are:

$$(R_p + n\,K^{\alpha}_{ac} + \tau\,L_p)\,I_{ac}(\tau) + $$
$$+ (n\,(K + K^{\beta}_{ac}) + \tau\,L^{bd}_{ac})\,I_{bd}(\tau) =$$
$$= n\,K^{\gamma}_{ac}\,I_{\gamma}\,\frac{1}{\tau} \tag{40}$$
$$(n\,(K + K^{\alpha}_{bd}) - \tau\,L^{ac}_{bd})\,I_{ac}(\tau) -$$
$$- (R_s + n\,K^{\beta}_{bd} + \tau\,L_s)\,I_{bd}(\tau) = 0.$$

where R_r is the reactivity resistance of the metadyne:

$$R_r^2 = n^2\,K_{\pi}\,K_{\sigma} \tag{44}$$

the roots τ_1 and τ_2 of the determinant are:

$$\tau_1\,,\ \tau_2 =$$
$$= \frac{-1}{2\,(L_p\,L_s - L^2)}\Big[L_p\,R_{\sigma} + L_s\,R_{\pi} + \tag{45}$$
$$+ n\,L\,(K_{\pi} - K_{\sigma}) \pm$$
$$\pm \sqrt{(\ \ )^2 - 4\,(L_p\,L_s - L^2)\,(R_{\pi}\,R_{\sigma} + R_r^2)}$$

where the squared parenthesis into the radical is supposed to contain the quantity written outside the radical.

Formula (45), in spite of the abreviated symbols, is more laborious to comment than the ones considered

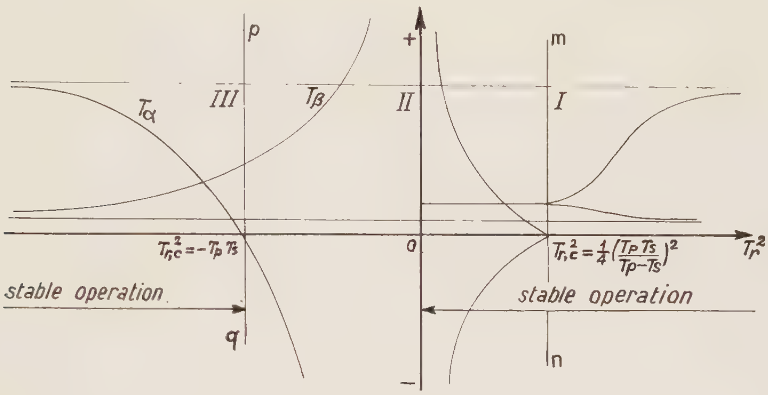


Fig. 7.

The coefficients K^{α}_{ac} and $K\,K^{\beta}_{bd}$ refer to windings 4 and 5 respectively (fig. 3); these coefficients are positive if said windings are stabilizers and negative for stimulators. Coefficients $L^{bd}_{ac} = L^{ac}_{bd}$ represent the mutual inductance between primary and secondary circuits and for the sake of simplicity we will cancel the suffixes.

The secondary variator winding has now a new zero mutual induction with winding 4 traversed by the primary current, but by means of transformer Tl the total mutual induction between the primary circuit and the secondary variator winding circuit is assumed to be reduced to zero.

In order to simplify scriptures we will put:

$$R_p + n\,K^{\alpha}_{ac} = R_{\pi}; \qquad R_s + n\,K^{\beta}_{ac} = K_{\sigma};$$
$$K + K^{\beta}_{ac} = K_{\sigma}; \qquad K + K^{\alpha}_{bd} = K_{\pi} \tag{41}$$

and we may refer to $n\,K^{\alpha}_{ac}$ and $n\,K^{\beta}_{bd}$ to «kinetic resistances».

Equations (50) may then be written as follows:

$$(R_{\pi} + \tau\,L_p)\,I_{ac}(\tau) + (n\,K_{\sigma} + \tau\,L)\,I_{bd}(\tau) =$$
$$= n\,K^{\gamma}_{ac}\,I_{\gamma}\,\frac{1}{\tau} \tag{42}$$
$$(n\,K_{\pi} - \tau\,L)\,I_{ac}(\tau) - (R_{\sigma} + \tau\,L_{\sigma})\,I_{bd}(\tau) = 0.$$

The determinant of the coefficients is:

$$-D = \tau^2[L_p\,L_s - L^2] + \tau[L_p\,R_{\sigma} + L_s\,R_{\pi} +$$
$$+ n\,L\,(K_{\pi} - K_{\sigma})] + R_{\pi}\,R_{\sigma} + R_r^2 \tag{43}$$

in the previous sections; following considerations may help simplifying.

The coefficient L is the sum of two others, say L' and L'', the coefficient L' being due to the primary flux of the metadyne and the coefficient L'' being due to the secondary. Assume that windings 4 and 5 (fig. 3) are both stabilizers and that windings 2 and 3 are either hypocompensators or amplifiers or non existing, then under the usual admissions in these books (i.e. assuming that n is positive when dextrorse and that the armature winding is dextrorse) the direction of the

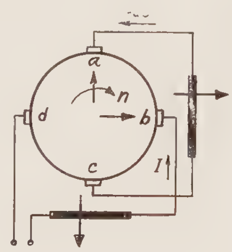


Fig. 8.

fluxes created by armature and the corresponding hypocompensator or amplifier, is given by the arrows inside the circle representing the armature in fig. 8 when the primary and secondary currents are positive; under same conditions the external arrows show the direction of the fluxes created by the stabilizers. It is thus apparent that L' are of opposite signs.

On the other hand the formula of the time constants of the transient currents show no difference of effect between the action of the primary and of the secondary

circuit; we can thus modify to some extent one of these circuits and yet obtain same values of time constants of the transient currents by an appropriate alteration of the other circuit.

Thus we may frequently reach equal absolute values for L' and L'' and hence have $L = 0$. If it is necessary a corrective transformer interlinking both circuits, primary and secondary, may be inserted.

Formula (45) is then and may be written as follows:

$$\tau_1\,,\ \tau_2 = \frac{-1}{2}\left[\frac{1}{T_\pi} + \frac{1}{T_\sigma} \pm \sqrt{\left(\frac{1}{T_\pi} - \frac{1}{T_\sigma}\right)^2 - \frac{4}{T_r^2}}\right] \tag{46}$$

where:

$$T_\pi = \frac{L_p}{R_\pi}\,; \qquad T_\sigma = \frac{L_s}{R_\sigma} \tag{47}$$

and then the comments are formally the same as the one developed in the previous section while the numerical values and practical results may differ widely.

Take for instance the case already illustrated by the diagrams of fig. 4.a and 4.b where $T_s = 10\ T_p$ and assume, for the sake of simplicity, that these may be only hypocompensators and amplifiers. Then only the right semiplane branches exist, for which stability of operation is always obtained. The minimum value of the larger time constant of the transient was found equal to:

$$T_q = 2\,T_p\left(L - \frac{T_p}{T_s} + \left(\frac{T_p}{T_s}\right)^2 - \ldots\right). \tag{48}$$

With the adoption of stabilizing windings said minimum becomes:

$$T_q = 2\,T_\pi\left(1 - \frac{T_\pi}{T_\sigma} + \left(\frac{T_\pi}{T_\sigma}\right)^2 - \ldots\right). \tag{49}$$

Yet we may arrange the metadyne windings for obtaining $T_\pi = \frac{1}{10}\ T_p$ and $T_\sigma = \frac{1}{2}\ T_s$ and hence have

$$T_q < \frac{1}{5}\ T_p = 0{,}02\ T_s \tag{50}$$

a remarkable result, which would reduce the overall apparent time constant of the system to a few percents of the time constant, of the circuit energized by the metadyne. Although imperfections in the construction of the machine reduce the effect of the reactivity, yet, practically it is so striking that, in shop incorrect parlance, it is said to « wipe out » the self induction of the energized circuit. The author is glad to remind here the enthusiastic interest arisen during the first oscillographic dynamic test in 1934, carried for the Italian Navy under the direction of Mr. B. Rossi, chief engineer of the Italian San Giorgio, a man giften by a very sensitive intuition and bright foresight.

The series windings considered in this section may be stimulators and the coefficients, K^α_{ac} and K^β_{bd}, negative; then R_π and R_σ may become zero and even negative. The time constants, T_π and T_σ may thus become negative and when one, at least, is negative, one or two time constants T_α and T_β are negative and there is building up. The critical values separating the domain of stable operation from the domain of unstable operation are determine by following equation:

$$\left(\frac{1}{T_\pi} + \frac{1}{T_\sigma}\right)^2 - \left(\frac{1}{T_\pi} - \frac{1}{T_\sigma}\right)^2 - \frac{4}{T_r^2} \tag{51}$$

which is of the second order for the speed.

Willful adoption of stimulators in a metadyne is rare, but undesired ones are frequent.

Let us complete this section by considering the asyptotic equations:

$$\begin{aligned} R_\pi I_{ac} + n\,K_\sigma I_{bd} &= n\,K^\gamma_{ac} I_\gamma \\ n\,K_\pi I_{ac} - R_\sigma I_{bd} &= 0. \end{aligned} \tag{52}$$

The determinant of the coefficients becomes zero when:

$$R_\pi R_\sigma + n^2 K_\pi K_\sigma = 0. \tag{53}$$

A comparison with equation (51) shows the identity of these two equations. R_π and R_σ are linear functions of speed, n and the discriminat, S, of equation (53), with respect speed n, is:

$$S = (R_p\,K^\beta_{bd} + R_s\,K^\alpha_{ac})^2 - 4\,K_\sigma\,K_\pi\,R_p\,R_s\,. \tag{54}$$

There are real values of critical speed as long as $S > 0$, hence whenever K_σ and K_π are of different sign. There are no real values of critical speed when $S < 0$ and then K_π and K_σ must be necessarily both positive or both negative.

We see thus that stable operation is possible with hypercompensators. If we particularly assume that $K_\pi = K_\sigma = -K$, in other words, if there are two hypercompensators with a degree of compensation equal to 200%, we have the same operation as if the metadyne had no compensators whatever and only the direction of currents changes. This property finds practical applications.

6.) Metadyne's reactivity as a stability factor.

The operation of a series excited dynamo, supposed inserted in the secondary circuit of the metadyne or in the circuit of the second dynamo of the arrangement fig. 2, may be represented by a kinetic resistance; the latter is positive or negative when the dynamo operates as a motor or as a generator correspondingly.

Let us assume that the series excited dynamo is operating as a generator; then the ohmic resistance R in equations (17) must be replaced by the sum of the ohmic resistance R and the kinetic resistance, say R_k, which is now negative. If then $R + R_k < 0$, time constant T in solutions (22) and (23) is negative and there is building up. As generally R is very small, even a moderate action of a recuperating series dynamo brings instability into the system of fig. 2 as br ngs instability into a system comprising a constant voltage, direct current or alternating current, network and a series dynamo supplied by it.

A series dynamo inserted into the secondary circuit of a simple reactive metadyne as the one schematically represented by fig. 1, will operate according solutions (24) and (24,1) where time constants T_α, T_β and T_q remain positive even for large values of negative kinetic resistance of the series excited dynamo. In fact roots τ_1 and τ_2 of equations (10) may now become positive because

$$T_s = \frac{L_s}{R_s + R_k} \tag{55}$$

is now negative when $R_k < -R_s$ and one of the roots becomes negative with a critical value of R_k deter-

mined by following equation:

$$n^2 K^2 + R_p (R_s + R_{k,s}) = 0 \tag{56}$$

deduced from (53). Hence instability is reached only when

$$R_{k,s} < -\frac{n^2 K^2}{R_p} - R_s . \tag{57}$$

Comparing inequalities (55) and (57) and remembering that $n K$ is about 50 times larger than R_p and usually at least two times larger than R_s, we may deduce that the reactive metadyne will preserve stability even when the stimulating action of the series dynamo becomes one hundred times larger than the one sufficient to bring instability in a constant voltage direct current or alternating current network.

This example illustrates another characteristic of the reactivity of the metadyne.

7.) Behavior of the current traversing a coil supplied by a reactive metadyne.

When it is desired to energize a coil with a predetermined direct current, say I, we may use a dynamo or a metadyne. When this coil is interlinked with a flux showing a periodic or a transient variation, there will be a periodic or a transient disturbance of the current traversing the coil; we intend to determine this disturbance and compare the behavior of the two systems, dynamo and coil or metadyne and coil.

Fig. 9 and 10 show the scheme of the two systems and it is assumed that the machines have same size and rotate at same speed.

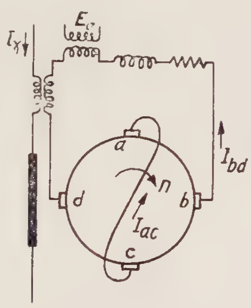


Fig. 9.

Taking into account a Heavisidean flux disturbance of pulsation ω, the equations determining the current are for a metadyne supply:

$$\left(R_p + L_p \frac{d}{dt}\right) I_{ac} + n K I_{bd} - n K^{\gamma}_{ac} I_\gamma = 0$$

$$n K I_{ac} - \left(R_s + L_s \frac{d}{dt}\right) I_{bd} + C(t)_0 = 0 \tag{58}$$

where $C(t)_0$ represents the e.m.f. induced into the coil by the Heavisidean flux disturbance.

The solution of equations (58) comprises obviously a constant term and a term function of time. The constant terms of the first member of these equations on one side, and the terms functions of time, on the other side, must give a zero sum separately. Let us decompose currents as follows:

$$I_{ac} = I_{ac,p} + I_{ac,q}; \quad I_{bd} = I_{bd,p} + I_{bd,q} \tag{59}$$

where the third index p indicates a direct current component, and index q a transient component; then equations (58) may be decomposed as follows:

$$R_p I_{ac,p} + n K I_{bd,p} - n K^{\gamma}_{ac} I_\gamma$$

$$n K I_{ac,p} - R_s I_{bd,p} = 0$$

$$\left(R_p + L_p \frac{d}{dt}\right) I_{ac,s}(t)_0 + n K I_{bd,s}(t)_0 = 0 \tag{60}$$

$$n K I_{ac,s}(t)_0 - \left(R_s + L_s \frac{d}{dt}\right) I_{bd,s}(t)_0 = - C(t)_0 .$$

The solution of the second pair of these equations is:

$$I_{bd,s}(t)_0 = \frac{C}{R_s + \dfrac{n^2 K^2}{R_p}} \cdot \left[1 + A e^{-\frac{t}{T_\alpha}} + B e^{-\frac{t}{T_\beta}}\right] \tag{61}$$

where T_α and T_β have same values as in formula (23) and where A and B are constants of integration.

If the external part of the secondary circuit of the metadyne is connected to a dynamo of same size, as fig. 10 shows, or to a constant voltage network, the transient current under same disturbance, is:

$$I(t)_0 = \frac{C}{R_s}\left[1 - e^{-\frac{t}{T_s}}\right] . \tag{62}$$

Comparing formula (61) and (62) it becomes apparent that when the metadyne is energizing the circuit the transient component due to the disturbance has an asymptotic value much smaller which may be only a few thousands of what it is when same circuit is energized by a dynamo (or a constant voltage network).

If the metadyne reactivity resistance R_r is brought to zero by a 100% degree compensator, then the factor outside the parenthesis of formula (61) becomes equal to the corresponding factor of formula (62).

If finally the reactivity resistance R_r, is given imaginary values, the avove mentioned factor of formula (61) may reach very high values.

In « metadyne combinators » same applications of properties are indicated.

Let us now consider a sinusoidal flux perturbance abruptly beginning at time $t = 0$, and decompose currents as follows:

$$I_{ab} = I_{ab,p} + I_{ab,r} + I_{ab,s};$$

$$I_{bd} = I_{bd,p} + I_{bd,r} + I_{bd,s} \tag{63}$$

where the the new third index s, indicates alternating current component. Then equations (58) may then be decomposed into the following ones:

$$R_p I_{ac,p} + n K I_{bd,p} = n K^{\gamma}_{ac} I_\gamma$$

$$n K I_{ac,p} - R_s I_{bd,p} = 0$$

$$(R_p + j\,\omega L_p)\, \bar{I}_{ac,r} + n K \bar{I}_{bd,r} = 0$$

$$n K \bar{I}_{ac,r} - (R_s + j\,\omega L_s)\, \bar{I}_{bd,r} = -\bar{C} \tag{64}$$

$$\left(R_p + L_p \frac{d}{dt}\right) I_{ac,s}(t)_0 + n K I_{bd,s}(t)_0 = 0$$

$$n K I_{ac,s}(t)_0 - \left(R_s + L_s \frac{d}{dt}\right) I_{bd,s}(t)_0 = - C(t)_0$$

obtained by equating separately the constant terms, the sinusoidal terms and the exponential terms assumed to have the real part of their exponents different from zero. Symbols $I_{ac,\,r}$ and $I_{bd,\,r}$ indicate the vectors of a Fresnel diagram corresponding to the asymptotic sinusoidal components.

The first two equations (60) give the direct current component and the second pair of equations give the asymptotic value of the alternating current component the permanent perturbation. The latter is represented by following vector:

$$\bar{I}_{bd,\,r} = \frac{-\bar{C}\,(R_p - j\,\omega\,L_p)}{(R_p + j\,\omega\,L_p)\,(R_s + j\,\omega\,L_s) + n^2\,K^2}$$

$$(65) \qquad \frac{\bar{C}}{R_s + j\,\omega\,L_s} \cdot \frac{1}{1 + \dfrac{n^2\,K^2}{(R_p + j\,\omega\,L_p)\,(R_s + j\,\omega\,L_s)}}$$

The first factor of the last formula is obviously the vector representing the perturbation in case of a dynamo supply according to scheme of fig. 10. The second factor is a dimensional and a complex number whose argument is positive. If we assume that the ohmic resistances R_p and R_s are very small with respect the corresponding inductances $\omega\,L_p$ and $\omega\,L_s$, so that we may neglect them, the numerical factor becomes real, equal to:

$$(66) \qquad \frac{1}{1 - \dfrac{n^2\,K^2}{\omega^2\,L_p\,L_s}} = \frac{1}{1 - \dfrac{R_r^2}{\omega^2\,L_p\,L_s}}$$

In most practical cases there is $n^2\,K^2 > \omega^2\,L_p\,L_s$ and then factor (66) has a modulus smaller than one, and the asymptotic value of the perturbance is smaller than the perturbance in case of a dynamo supply.

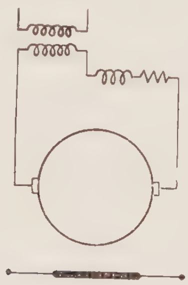


Fig. 10.

By addition of series stator windings on the metadyne the square of the reactivity resistance, R_r^2, depending on said windings in the known manner may take any positive or negative value. Formula (66) remains valid as long as the mutual induction between primary and secondary circuit is nil. Thus if there is a compensating winding of 100% degree of compensation, the perturbance becomes the same no matter what the supply is, dynamo or metadyne; if R_r^2 takes values for which the modulus of the denominator or the second factor of (65) is smaller than one, the perturbance is larger with metadyne supply.

Thus we arrive at similar conclusions as in case of a Heavisideean disturbance, with constant asymptotic value; we may generalize these conclusions for any disturbance.

8.) Action of vinculators comprising resistance and inductance.

For the sake of simplicity we consider only three stator windings, as the shceme of fig. 11 shows: a secondary variator and two vinculators traversed by currents I_δ and I_ε respectively.

Let us close switch S when current I_γ is established; following equations represent the transient operation:

$$\left(R_p + L_p \frac{d}{dt}\right) I_{ac}(t)_0 + n\,K\,I_{bd}(t)_0 + {}$$
$$+ L_p^\delta \frac{d}{dt} I_\delta(t)_0 = n\,K_{ac}^\gamma\,I_\gamma(t)_0$$

$$(67) \qquad n\,K\,I_{ac}(t)_0 - \left(R_s + L_s \frac{d}{dt}\right) I_{bd}(t)_0 -$$
$$- L_s^\varepsilon \frac{d}{dt} I_\varepsilon(t)_0 = 0$$

$$- L_p^\delta \frac{d}{dt} I_{ac}(t)_0 - \left(R_\delta + L_\delta \frac{d}{dt}\right) I_\delta(t)_0 = 0$$

$$- L_s^\varepsilon \frac{d}{dt} I_{bd}(t)_0 - \left(R_\varepsilon + L_\varepsilon \frac{d}{dt}\right) I_\varepsilon(t)_0 = 0$$

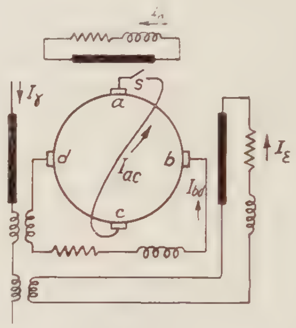


Fig. 11.

where the symbols L_p^δ, L_s^ϵ, R_δ, L_δ, R_ϵ, L_ϵ, have obvious significance. Applying Laplace transform and eliminating $I_\delta(\tau)$ and $I_\varepsilon(\tau)$, we obtain following two equations with only two variables:

$$I_{ac}(\tau)\,[R_p\,R_\delta + \tau\,(L_p\,R_\delta + L_\delta\,R_p) -$$
$$- \tau^2\,((L_\beta^\delta)^2 - L_\delta\,L_p)] + n\,K\,(R_\delta + \tau\,L_\delta)$$
$$I_{bd}(\tau) = n\,K_{ac}^\gamma\,I_\gamma \frac{R_\delta + \tau\,L_\delta}{\tau}$$

$$(68) \qquad (R_\varepsilon + \tau\,L_\varepsilon)\,n\,K\,I_{ac}(\tau) -$$
$$- [R_s\,R_\varepsilon + \tau\,(L_s\,R_\varepsilon + L_\varepsilon\,R_s) -$$
$$- \tau^2\,((L_s^\varepsilon)^2 - L_\varepsilon\,L_s)]$$
$$I_{bd}(\tau) = 0.$$

The determinant of the coefficients is:

$$(69) \qquad -D = [\quad][\quad] + n^2\,K^2\,(R_\delta + \tau\,L_\delta) \cdot$$
$$\cdot\,(R_\varepsilon + \tau\,L_\varepsilon)$$

where the quantity braced by the rectangular parenthesis are the same as in equations (68). The roots

of equation:

$$D = 0 \tag{70}$$

are laborious to write and to comment.

The action of the two vinculators may represent approximately the action of the iron losses due to hysteresis and Foucault eddy currents, and hence the fact that the flux traversing the laminations does not follow immediately the variation of the resultant ampere-turns. The corresponding values of R_δ, R_ε, L_δ, L_ε, and L_p^δ and L_s^ε can be approximately determined by test, to be later on reported. Let us divide both member of (70) by $R_\delta\, R_\varepsilon$; we obtain:

$$\begin{aligned} &[R_p + \tau (L_p + R_p T_\delta) - \tau^2 (L_p^\delta T_\delta - L_p T_\delta)] \cdot \\ &\cdot [R_s + \tau (L_s + R_s T_\varepsilon) - \tau^2 \\ &(L_s^\varepsilon T_\varepsilon - L_s T_\varepsilon)] + n^2 K^2 (1 + \tau T_\delta) \cdot \\ &\cdot (1 + \tau T_\varepsilon) = 0. \end{aligned} \tag{71}$$

Tests show that T_δ and T_ε are very small with good laminations; hence the two roots of (71) are grouped, on the Gauss plane in the vicinity of the roots of equation:

$$(R_p + \tau L_p)(R_s + \tau L_s) + n^2 K^2 = 0. \tag{72}$$

The roots of this equation were discussed in the previous sections.

We may thus deduce that the imperfection of the iron magnetic circuit will modify the results indicated in the previous chapters but to a small extent.

The analysis here above developed may be readily extended to the case of a metadyne provided with any series winding arrangement by substituting R_r^2, R_π and R_σ to $n^2 K^2$, R_p and R_s respectively in the equations of this section under the assumption that $L = 0$. We may then say that if the two roots of equation:

$$(R_\pi + \tau L_p)(R_\sigma + \tau L_s) + R_r^2 = 0 \tag{73}$$

are well within the borders of the stability domain in the Gauss plane, the imperfections of the iron magnetic circuit will quite probably not bring instability.

We may also note that in all previous sections the formulas are symmetric with respect R_π and R_σ and with respect L_p and L_s and we may deduce that for the slightly damping interference of the iron losses, similar winding have similar final action no matter the circuit in which there are inserted.

9.) Permanent oscillations.

Let us consider the arrangement indicated by the scheme of fig. 3, assuming that the external circuit comprises only resistance and self induction. We may set windings 2, 3, 4, and 5 so as to satisfy following relations derived from formula (45).

$$\begin{aligned} &L_p R_\sigma + L_s R_\pi + n L (K_\pi - K_\sigma) = 0 \\ &4 (L_p L_s - L^2)(R_\pi R_\sigma + R_r^2) > 0. \end{aligned} \tag{74}$$

If, under these conditions, the secondary variator winding current, I_γ, is given an impulse at the end of which this current is brought to zero, we shall have a permanent operation with sinusoidal currents whose pulsation, ω_z, is defined by:

$$\omega_z^2 = \frac{(R_\pi R_\sigma + R_r^2)}{L_p L_s - L^2}. \tag{75}$$

With an occasional insertion of corrective resistors and inductances into the primary and the secondary circuit, and with the possibility of setting the value of the speed, n, we dispose of 5 variables which can be given practically any value comprised between two limits depending of the size of the machine. Thus the value of the pulsation ω_z may be modified within a wide range.

For instance, if $K_\pi = K_\sigma$ and $R_\pi = - R_\sigma \dfrac{L_p}{L_s}$ there will be permanent oscillations for any value of the speed for which $R_r^2 > |R_\pi R_\sigma|$. If $R_\pi = R_p$; $R_\sigma = R_s$, L is zero and there cannot be permanent oscillations, except if a transformer interlinks further the primary and the secondary circuit as shown by fig. 12.

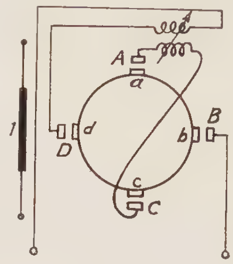


Fig. 12.

If $L_p R_\sigma + L_\sigma R_\pi \neq 0$ and $K_\pi \neq K_\sigma$, there is only one value of the speed for which there are permanent oscillations for definite values of R_π, R_σ, L_p, L_s, and L; if the latter can be modified with continuity by means of a transformer with movable members of its magnetic circuit, a futher independent variable is at our disposal for controlling the pulsation ω_z.

10.) Shunt windings.

The diagram of fig. 13 shows a main metadyne, I with the terminals A, B, C, D, of the possible series windings located opposite the corresponding brushes a, b, c, d symbol already used in the previous volumes permitting to simplify the scheme by assuming the existance of series windings without representing them. Outside of the possible series windings there are two shunt windings a primary, 6, and a secondary, 7, both

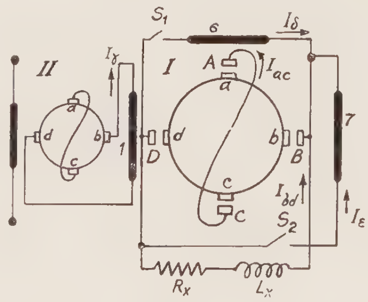


Fig. 13.

connected across the secondary terminals B and D, traversed by currents I_δ and I_ε respectively. A secondary variator winding, l is shown energized by an auxiliary metadyne, II: this arrangement, with adequate

stator windings on metadyne II, permits to admit that current I_γ is pratically independent from variations of the fluxes of metadyne I interlinked with variator l. Such arrangement is used by the author for precise control.

A heavisidean current $I_\gamma\,(t)_0$, reaching abruptly the constant value I_γ at $t = 0$, with all currents nil for $t > 0$, gives rise to a transient determined by following Laplace transformed equation:

$$
(76)\qquad
\begin{aligned}
&(R_\pi + \tau\, L_p)\, I_{ac}(\tau) + (n\, K_\sigma + L\,\tau)\, I_{bd}(\tau) + \\
&+ \tau\, L_p^\delta\, I_\delta(\tau) = n\, K_{ac}^\gamma\, I_\gamma\, \frac{1}{\tau} + n\, K_{ac}^\varepsilon\, I_\varepsilon(\tau) \\
&(n\, K_\pi - \tau\, L)\, I_{ac}(\tau) - (R_\sigma + \tau\, L_s)\, I_{bd}(\tau) + \\
&+ n\, K_{bd}^\delta\, I_\delta(\tau) - \tau\, L_s^\varepsilon\, I_\varepsilon(\tau) = 0 \\
&- L_p^\delta\, \tau\, I_{ac}(\tau) - (R_\delta + \tau\, L_\delta)\, I_\delta(\tau) + \\
&+ (R_x + \tau\, L_x)\, I_{bd}(\tau) = 0 \\
&- L_s^\varepsilon\, \tau\, I_{bd}(\tau) - (R_\varepsilon + \tau\, L_\varepsilon)\, I_\varepsilon(\tau) + \\
&+ (R_x + \tau\, L_x)\, I_{bd}(\tau) = 0
\end{aligned}
$$

with obvious significance of the usual symbols. The determinant of the coefficients is an polynomial of the fourth degree with respect to τ, whose roots are too laborious to comment.

The research of a possible oscillatory asymptotic operation leads us to a system of equations that may be immediately derived from (76)

$$
(76')\qquad
\begin{aligned}
&(R_\pi + j\,\omega\, L_p)\, \bar I_{ac} + (n\, K_\sigma + j\,\omega\, L)\, \bar I_{bd} + \\
&\quad + j\,\omega\, L_p^\delta\, \bar I_\delta = n\, K_{ac}^\varepsilon\, \bar I_\varepsilon \\
&(n\, K_\pi - j\,\omega\, L)\, \bar I_{ac} - (R_\sigma + j\,\omega\, L_s)\, \bar I_{bd} + \\
&\quad + n\, K_{bd}^\delta\, \bar I_\delta - j\,\omega\, L_s^\varepsilon\, \bar I_\varepsilon = 0 \\
&j\,\omega\, L_p^\delta\, \bar I_{ac} + (R_\delta + j\,\omega\, L_\delta)\, \bar I_\delta - \\
&\quad - (R_x + j\,\omega\, L_x)\, \bar I_{bd} = 0 \\
&j\,\omega\, L_s^\varepsilon\, \bar I_{bd} + (R_\varepsilon + j\,\omega\, L_\varepsilon)\, \bar I_\varepsilon - \\
&\quad - (R_x + j\,\omega\, L_x)\, \bar I_{bd} = 0
\end{aligned}
$$

sufficient for originating the oscillations; therefore no term in I_γ figures in them.

In order to determine the possible values of a real pulsation ω, we are led again to equate to zero the determinant of the coefficients of said equations, a polynomial of the fourth degree with respect to ω.

The dynamic equations are simplified if we consider each shunt winding separately and a secondary compensating winding with 100% compensation degree. Let first switch S 1 closed and switch S 2 openi n fig. 13 The. Laplace transform of the corresponding dynamyc equations is then:

$$
(77)\qquad
\begin{aligned}
&(R_\pi + \tau\, L_p)\, I_{ac}(\tau) + \tau\, L\, I_{bd}(\tau) + \tau\, L_p^\delta\, I_\delta(\tau) = \\
&\quad = n\, K_{ac}^\gamma\, I_\gamma(\tau) \\
&(n\, K_\pi - \tau\, L)\, I_{ac}(\tau) - (R_\sigma + \tau\, L_s)\, I_{bd}(\tau) + \\
&\quad + n\, K_{bd}^\delta\, I_\delta(\tau) = 0 \\
&- \tau\, L_p^\delta\, I_{ac}(\tau) + (R_x + \tau\, L_x)\, I_{bd}(\tau) - \\
&\quad - (R_\delta + \tau\, L_\delta)\, I_\delta(\tau) = 0.
\end{aligned}
$$

Primary resistance, R_p, is generally very small; we may set $R_\pi = 0$ and reduce the determinant of the coefficients to following value:

$$
\tau \begin{vmatrix}
L_p & L & L_p^\delta \\
n\, K_\pi - \tau\, L & -(R_\sigma + \tau\, L_s) & n\, K_{bd}^\delta \\
-\tau\, L_p^\delta & R_x + \tau\, L_x & -(R_\delta + \tau\, L_\delta)
\end{vmatrix}
$$

showing a polynomial of the second degree as factor. In order to simplify this polynomial, let us further assume that there is no secondary stabilizing winding. Then $L = 0$ and the second degree polynomial, equated to zero, simplifiers as follows when divided by L_s:

$$
(77')\qquad
\begin{aligned}
&\tau^2 \left[L_\delta\, L_p - (L_p^\delta)^2\right] + \tau \left[\frac{L_p}{L_s}(R_\delta\, L_s + L_\delta\, R_\sigma) + \frac{L_x}{L_s}(L_p^\delta\, n\, K_\pi - L_p\, n\, K_{bd}^\delta) - \frac{L_p^\delta}{L_s} R_\sigma\, L_p^\delta\right] + \\
&\qquad + \frac{L_p}{L_s}\left[R_\sigma\, R_\delta + R_x\, n\,(K_\pi - K_{bd}^\delta)\right] = 0.
\end{aligned}
$$

where the unknowns are vectors $\bar I_{ac}$, $\bar I_{bd}$, $\bar I_\delta$ and $\bar I_\varepsilon$ referred, as usually, to a Fresnel diagram on a Gauss plane. These equations (76') are written under the assumption that a temporary impulse of current I_γ is

The borders of the stability domain are defined by following relations:

$$
(78)\qquad
\begin{aligned}
&4\, L_p\left[R_\sigma\, R_\delta + R_x\, n\,(K_\pi - K_{bd}^\delta)\right]\left[L_d\, L_p - (L_p^\delta)^2\right] > \left[L_p\,(R_\delta\, L_s + L_\delta\, R_\sigma) + L_x\,(L_p^\delta\, K_\pi - L_p\, K_{bd}^\delta)\, n - \right. \\
&\qquad \left. - (L_p^\delta)^2\, R_\sigma\right]^2 \\
&L_p\,(R_\delta\, L_s + L_\delta\, R_\sigma) + L_x\,(L_p^\delta\, K_\pi - L_p\, K_{bd}^\delta)\, n - (L_p^\delta)^2\, R_\sigma = 0
\end{aligned}
$$

and by following equation:

$$
(79)\qquad R_\sigma\, R_\delta + R_x\, n\,(K_\pi - K_{bd}^\delta) = 0
$$

both equations are of the first degree with respect the rotational speed n. The value of the critical speed yielded by (79):

$$(80) \qquad n = R_\delta \frac{R_\sigma}{R_x} \cdot \frac{1}{K_{bd}^{\delta} - K_\pi}$$

may be compared to the value of the building up speed of same machine when operates as a shunt excited dynamo by opening the primary circuit, i.e. with value:

$$n = (R_\delta + R_m) \frac{1}{K_{bd}^{\delta}}$$

where R_m is the resistance of the armature of the metadyne plus the resistance of the secondary compensator. Taking into account that R_m and K_π are very small when compared to R_δ and K_{bd}^{δ} respectively, and that R_σ is not very different generally from R_x, we may derive following deduction:

When the metadyne has secondary 100% compensator, a negligible primary resistance and is deprived of secondary stabilizing winding, these are generally the conditions met in an amplidyne, the operation becomes unstable practically at the same conditions for which the same machine with open primary circuit, operating as a excited dynamo builds up (fig. 14).

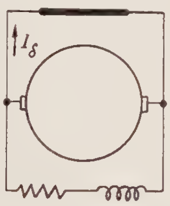


Fig. 14.

Switches S 1 and S 2 will now be assumed originally open and then only S_2 closed bringing into operation the shunt winding 7 alone. We shall further assume that there is are no secondary stabilizing windings. Under these conditions there is no mutual induction between the secondary circuit of the metadyne and winding 7 and the Laplace transform of the dynamic equations may be written as follows:

$$(81) \qquad \begin{aligned} &(R_p + \tau L_p) I_{ac}(\tau) - n K_{ac}^{\varepsilon} I_\varepsilon(\tau) = n K_{ac}^{\gamma} I_\gamma(\tau) \\ &n K I_{ac}(\tau) - (R_s + \tau L_s) I_{bd}(\tau) = 0 \\ &n K I_{ac}(\tau) - (R_m + \tau L_m) I_{bd}(\tau) - \\ &\qquad - (R_\varepsilon + \tau L_\varepsilon) I_\varepsilon(\tau) = 0 \end{aligned}$$

where R_m and L_m refer to the resistance and self inductance of this section of the secondary circuit which is part of the metadyne.
The determinant of the coefficients is:

$$(81') \qquad D = \begin{vmatrix} R_p + \tau L_p & 0 & - n K_{ac}^{\varepsilon} \\ n K & -(R_s + \tau L_s) & 0 \\ n K & -(R_m + \tau L_m) & -(R_\varepsilon + \tau L_\varepsilon) \end{vmatrix}$$

We consider the practically very probable case that R_m and L_m are proportional to the resistance R_x and self inductance L_x of the external part of the secondary circuit, i.e. that there is:

$$(82) \qquad \begin{aligned} R_x &= k R_m \\ L_x &= k L_m \end{aligned}$$

and then the determinant is decomposed as follows:

$$D = -(R_m + \tau L_m) \begin{vmatrix} R_p + \tau L_p & 0 & - n K_{ac}^{\varepsilon} \\ n K & k + 1 & 0 \\ n K & 1 & -(R_\varepsilon + \tau L_\varepsilon) \end{vmatrix}$$

The determinant figuring in this formula is a polynomial of the second degree having following roots:

$$\tau_1, \tau_2 = \frac{-(L_p R_\varepsilon + L_\varepsilon R_p) \pm \sqrt{\left(\quad \right)^2 - 4 L_p L_\varepsilon \left(R_p R_\varepsilon - n^2 K K_{ac}^{\varepsilon} \dfrac{k}{k+1} \right)}}{2 L_p L_\varepsilon}.$$

Note that K_{ac}^{ϵ} is positive when winding 7 tends to increase the primary current and the secondary voltage, a generally desired action. Note further that R_ϵ and $n K_{ac}^{\epsilon}$ are of the same order and that R_p is generally less than 1% of $n K$, hence a small value of n suffices to bring instability and building up, the border of stability being defined by equation:

$$(83) \qquad n^2 K K_{ac}^{\epsilon} \frac{k}{k+1} = R_p R_\varepsilon .$$

One may argue as follows: suppose that the secondary terminals of the metadyne are short circuited, then current $I_\varepsilon = 0$; how can be then instability? There will be true stability in this case for whatever value of the speed, but there is no contradiction with the previous arguments because formula (83) shows that the value of the critical speed is also a function on k which decreases when the external part of the secondary circuit decreases and becomes zero when the metadyne terminals are short circuited, the value of the critical speed tending to infinite.

We may thus conclude that under the practically probable conditions expressed by equation:

$$\frac{R_x}{R_m} = \frac{L_x}{L_m}$$

the addition of shunt winding 7 will bring instability even for a relatively small number of developed ampere turns when the reactivity of the metadyne is reduced to zero and there are no stabilizing windings.

The general case leads to a differential equation of a degree, too high for an easy comment. In order to obtain some information we revert to the equations of the asymptotic direct current operation:

$$(84) \qquad \begin{aligned} &R_\pi I_{ac} + n K_\sigma I_{bd} = n K_{ac}^{\gamma} I_\gamma + n K_{ac}^{\varepsilon} I_\varepsilon \\ &n K_\pi I_{ac} - R_\sigma I_{bd} = - n K_{bd}^{\delta} I_\delta \\ &R_x I_{bd} - R_\delta I_\delta = 0 \\ &R_x I_{bd} - R_\varepsilon I_\varepsilon = 0 . \end{aligned}$$

Coefficient K_{ac}^{ε} is positive when a positive current I_ε tends to increase the action of I_γ; coefficient K_{bd}^{δ} is

positive when a positive current I_δ tends to increase the action of I_{ac}. Eliminating I_δ and I_ε we have:

$$R_\pi I_{ac} + \left[n K_\sigma - n K_{ac}^{\varepsilon} \frac{R_x}{R_\varepsilon} \right] I_{bd} = n K_{ac}^{\gamma} I_\gamma$$

$$n K_\pi I_{ac} + \left[n K_{bd}^{\delta} \frac{R_x}{R_\delta} - R_\sigma \right] I_{bd} = 0.$$

The determinant of the coefficients is:

$$D \quad R_\pi \left[n K_{bd}^{\delta} \frac{R_x}{R_\delta} - R_\sigma \right] + n^2 K_\pi \left[K_{ac}^{\varepsilon} \frac{R_x}{R_\varepsilon} - K_\sigma \right].$$

The two roots of the determinant equated to zero are:

$$(85) \qquad n_1 , n_2 = \frac{- R_\pi K_{bd}^{\delta} \dfrac{R_x}{R_\delta} \pm \sqrt{(\quad)^2 + 4 K_\pi \left(K_{ac}^{\varepsilon} \dfrac{R_x}{R_\varepsilon} - K_\sigma \right) R_\pi R_\sigma}}{2 K_\pi \left(K_{ac}^{\varepsilon} \dfrac{R_x}{R_\varepsilon} - K_\sigma \right)}$$

Let us first consider winding 7 operating and winding 6 disconnected; in this case the roots of the determinant, divided by the nominal speed n_N of the metadyne, are:

$$\frac{n_1}{n_N}, \frac{n_2}{n_N} = \pm \sqrt{\frac{R_\pi R_\sigma}{K_\pi \left(K_{ac}^{\varepsilon} \dfrac{R_x}{R_\varepsilon} - K_\sigma \right) n_N^2}}.$$

By multiplying the numerator and the denominator of the fraction under the radical sign by the square of the nominal durrent of the metadyne, I_N^2, we find that for usual metadynes, the quantity:

$$(86) \qquad K_{ac}^{\varepsilon} \frac{R_x}{R_\varepsilon} - K_\sigma$$

is negative. In fact $n_N K I_N$ is the voltage induced between two electrical diametrically opposite brushes when the other pair is traversed by the nominal current, generally 3 times the nominal voltage V_N; similarly $n_N K_{ac}^{\varepsilon} \dfrac{V_N}{R_\varepsilon}$ is generally equal to V_N and hence the non dimensional quantity:

$$\frac{n_N K_{ac}^{\varepsilon}}{R_\varepsilon}$$

is generally equal to 1. On the other hand $R_x I_N$ is obviously equal or smaller than V_N, therefore (86) is generally negative and we may deduce that quite probably there are no real values of speed n for which the operation is unstable.

Note that quantity (86) may become as small as we like, provided $K_\sigma \neq 0$; hence the critical speed may become as large as we like. But if $K_\sigma = 0$, same argumentation leads to small values of the critical speed with respect the nominal speed, n_N, meaning probable instability.

If now winding 6 operates and 7 is idle, the roots of the determinant divided by n_N, become, in case $K_\sigma \neq 0$:

$$\frac{n_1}{n_N}, \frac{n_2}{n_N} = \frac{R_\pi R_x K_{bd}^{\delta} n_N}{2 R_\delta n_N^2 K_\pi K_\sigma} \pm \sqrt{(\quad)^2 - \frac{R_\pi R_\sigma}{n_N^2 K_\pi K_\sigma}}.$$

This formula gives imaginary critical speeds even for values of K_{bd} corresponding to an astatic shunt winding 6, in fact in this it is easy to see that $K_{bd}^{\delta} \, n_N \dfrac{1}{R_\delta} = 1$. If moreover $K_\sigma = 0$, then the root of the determinant becomes:

$$n = \frac{R_\delta}{K_{bd}^{\delta}} \, \frac{R_\sigma}{R_x}$$

practically same as the critical speed of an equivalent shunt excited dynamo.

Finally considering both shunt windings operating and $K_\sigma \neq 0$, and applying on the complete formula (85) same arguments as above developed, we find that a skillful determination of stator windings may render quantity (86) small enough as to have a ratio of the critical values with respect to the nominal value safely high, or even may give imaginary values for the critical speed leading us to admit that the machine will never leave the stability domain.

Thus the investigation of the asymptotic operation not only confirms the deduction drawn considering special cases of dynamic equations, but it appears to show that with a reactive metadyne it is possible to have stability with a much wider range of possible shunt windings, than with an areactive one or with an equivalent dynamo.

11.) Insertion of capacitors into the circuits.

Capacitors may be inserted into armature circuits while operating with alternating currents. Capacitors in stator winding circuits, may be inserted even when operating with direct current. Thus fig. 15 shows a scheme comprising a capacitor, C_s, inserted into the secondary circuit and another, C_δ, inserted into the circuit of the primary vinculator winding, 8; when

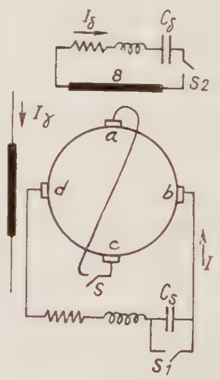


Fig. 15.

operating with direct current, e.m. forces may be induced into the vinculator by a transient component of the primary current, and the presence of capacitor C_δ influences said transients. Note that an appropriate choice of a large number of turns of thin wire for the vinculator, will permit to use a relatively small capacitor.

The secondary variator winding, 1, is supposed energized by a reactive metadyne and hence current I_γ is practically independent of disturbances due to transient metadyne fluxes.

Let us first consider the action of C_s alone and assume that switches S 1 and S 2 are open; then the Laplace transforms of the corresponding dynamic equations are, under the usual original conditions of the currents, as follows:

$$(87)\quad \begin{aligned} &(R_p + \tau L_p)\, I_{ac}(\tau) + n K I_{bd}(\tau) = n K_{ac}^{\gamma} I_\gamma \frac{1}{\tau} \\ &n K I_{ac} - \left(R_s + \tau L_s + \frac{1}{\tau C_s} \right) I_{bd}(\tau) = 0. \end{aligned}$$

Mutual induction between secondary winding 1, traversed by I_γ, and other electrical circuits of the metadyne are here omitted, winding 1 being supposed energized by a very reactive metadyne. Hence:

$$(88)\quad \frac{I_{bd}(\tau)}{I_\gamma} = n K_{ac}^{\gamma} \frac{1}{(R_p + \tau L_p)(R_s \tau + \tau^2 L_s + S_s) + \tau n^2 K^2}$$

where $S_s = 1/C_s$ is the elastance corresponding to capacity C_s. The denominator of (88), equated to zero, gives:

$$(89)\quad \begin{aligned} &\tau^3 L_p L_s + \tau^2 (R_p L_s + R_s L_p) + \\ &+ \tau (R_p R_s + L_p S_s + n^2 K^2) + R_p S_s = 0 \end{aligned}$$

an equation of the third degree in its most general form, hence leading to laborious expression of the roots.

Renouncing to a methodical comment, let us, at least, consider some special cases: if we assume that the time constant, T_p, of the primary circuit is very small, and divide equation (89) by R_p,

$$(90)\quad \begin{aligned} &\tau^3 L_s T_p + \tau^2 [L_s + R_s T_p] + \\ &+ \tau \left[R_s + S_s T_p + n^2 K^2 \frac{1}{R_p} \right] + S_s = 0 \end{aligned}$$

it becomes apparent that we may admit the three of equation (89) to be located, on the Gauss plane, in the vicinity of the two roots of following equation:

$$(91)\quad L_s \tau^2 + \left[R_s + n^2 K^2 \frac{1}{R_p} \right] \tau + S_s = 0$$

say

$$(92)\quad \tau_1, \tau_2 = \frac{- \left[R_s + n^2 K^2 \dfrac{1}{R_p} \right] \pm \sqrt{[\quad]^2 - 4 L_s S_s}}{2 L_s}.$$

The parenthesis into the radical bracing same quantity as the one outside of the radical.

From inspection of (92) we may deduct that, generally, even a low real reactivity resistance of the metadyne, keeps the operation stable and the transients terms non oscillatory; and reversely imaginary reacticity resistance of low modulus brings instability oscillations, and building up.

The above developed formulas remain formally the same in case series stator windings are added provided R_p, R_s, and $n^2 K^2$ are replaced by R_π, R_σ and n^2 $(K + K_{ac}^{ac})(K + K_{bd}^{bd}) = R_r^2$, respectively and provided that the mutual induction, L, between primary and secondary metadyne circuits, is, or is caused to be, nil.

If we now assume that the Capacity C_s is very large, hence S_s very small, we may admit that the three roots of (89) are located in the vincinity of the origin of the Gauss plane and of the two roots of the familiar equation:

$$(93)\quad \begin{aligned} &\tau^2 L_p L_s + \tau (R_\pi L_s + R_\sigma L_p) + R_\pi R_\sigma + \\ &+ R_r^2 = 0 \end{aligned}$$

whenever $L = 0$. We are then led to similar comments as developed in the first sections of this chapter I.

The second member of first equation of system (87) shows that we have admitted a Heavisidean form for the secondary variator current with constant value, I_γ, for $t > 0$.

We could instead admit a sinusoidal, or generally, a periodic form for same current; the transient terms will keep same exponentials. Thus the arguments developed in this section apply as well when metadyne generates direct or periodic currents.

Same remark is readily extended to all other cases examined in this Chapter.

Let us now close both switches, S 1 and S 2 of the scheme of fig. 15. The Laplace transform of the equations satisfied by transient components of the currents are:

$$\begin{aligned} &(R_\pi + \tau L_p)\, I_{ac}(\tau) + n K_\sigma I_{bd}(\tau) + \tau L_p^\delta I_{bd}(\tau) = \\ &\qquad = n K_{ac}^{\gamma} I_\gamma(\tau) \\ &n K_\pi I_{ac}(\tau) - (R_\sigma + \tau L_s)\, I_{bd}(\tau) + n K_{bd}^\delta I_\delta(\tau) = 0 \\ &- \tau L_p^\delta I_{ac}(\tau) - \left(R_\delta + \tau L_\delta + \frac{S_\delta}{\tau} \right) I_\delta(\tau) = 0. \end{aligned}$$

These equations are written under the assumption that $L = 0$, that the values of currents I_{ac}, I_{bd} and I_δ are 0 for $t < 0$, that current I_γ is inaffected by variations of metadyne fluxes and that it is an arbitrary function of time. Eliminating $I_\delta(\tau)$, we obtain:

$$\begin{aligned} &[\tau^3 (L_p L_\delta - (L_p^\delta)^2 + \tau^2 (L_p R_\delta + L_\delta R_\pi) + \\ &\quad + \tau (L_p S_\delta + R_\pi R_\delta) + R_\pi S_\delta]\, I_{ac}(\tau) + \\ &\quad + n K_\sigma (R_\delta \tau. + \tau^2 L_\delta + S_\delta)\, I_{bd}(\tau) = \\ &\quad = n K_{ac}^{\gamma} (\tau^2 L_\delta + \tau R_\delta + S_\delta)\, I_\gamma(\tau) \quad ; \\ &[\tau^2 n (K_\pi L_\delta - L_p^\delta K_{bd}^\delta) + \tau R_\delta n K_\pi + n K_\pi S_\delta]\, I_{ac}(\tau) + \\ &\quad - [\tau^3 L_s L_\delta + \tau^2 (L_\delta R_\sigma + L_s R_\delta) + \\ &\quad + \tau (L_s S_\delta + R_\sigma R_\delta) + R_\sigma S_\delta]\, I_{bd}(\tau) = 0. \end{aligned}$$

We are thus led to a determinant of the coefficients having the form of a polynomial of the sixth degree, more laborious to investigate than in any of the previous cases.

12.) Diagonal vinculator.

The « diagonal » vinculator is interlinked with both armature fluxes, the primary and the secondary, as shown schematically by windings 8 and 9 in fig. 16. These windings are indicated directly connected together without any external element inserted in their circuit in order to easily recognize their particular action.

Then Laplace transform of the dynamic equations under the usual boundary conditions and assumptions, are as follows:

$$(R_\pi + \tau L_p)\, I_{ac}(\tau) + (n K_\sigma + \tau L)\, I_{bd}(\tau) + \tau L^\delta_p\, I_\delta(\tau)$$
$$= n K^\gamma_{ac}\, I_\gamma(\tau) + n K^\delta_{ac}\, I_\delta(\tau)$$
$$(n K_\pi - \tau L)\, I_{ac}(\tau) - (R_\sigma + \tau L_s)\, I_{bd}(\tau) =$$
$$= \tau L^\delta_s\, I_\delta(\tau) - n K^\delta_{bd}\, I_\delta(\tau)$$
$$\tau L^\delta_p\, I_{ac}(\tau) - \tau L^\delta_s\, I_{bd}(\tau) = (R_\delta + \tau L^\delta_\delta)\, I_\delta(\tau) \quad 0.$$

Eliminating $I_\delta(\tau)$ we obtain two equations which divided by the resistance R_δ of the vinculator circuit yield:

$$I_{ac}(\tau)\left[\tau^2 L_p \frac{L^\delta_\delta}{R_\delta} + \tau\left(L_p + R_\pi \frac{L^\delta_s}{R_\delta} + n K^\delta_{ac} \frac{L^\delta_p}{R_\delta}\right) + R_\pi\right] +$$
$$+ I_{bd}(\tau)\left[\tau^2 L \frac{L^\delta_s}{R_\delta} + \tau\left(L + n K_\sigma \frac{L^\delta_\delta}{R_\delta} + n K^\delta_{ac} \frac{L^\delta_s}{R_\delta}\right)\right] = n K^\gamma_{ac}\, I_\gamma(\tau)\left[1 + \tau \frac{L^\delta_\delta}{R_\delta}\right]$$
$$I_{ac}(\tau)\left[-\tau^2 L \frac{L^\delta_p}{R_\delta} + \tau\left(n K_\pi \frac{L^\delta_\delta}{R_\delta} - L - n K^\delta_{bd} \frac{L^\delta_p}{R_\delta}\right)\right] +$$
$$- I_{bd}(\tau)\left[\tau^2 L_s \frac{L^\delta_\delta}{R_\delta} + \tau\left(L_s + R_\sigma \frac{L^\delta_\delta}{R_\delta} + n K_{bd} \frac{L^\delta_s}{R_\delta}\right) + R_\sigma\right] = 0.$$

The determinant of the coefficient is of the fourth degree.

We may note that the time constant of the vinculator is small, generally, as compared to time constants, T_p and T_s, of the primary and the secondary circuits; in any case, we may set R_δ as large as we want and render thus the numerical value of the coefficients of τ^2 of the coefficients $R_\pi \dfrac{L^\delta_s}{R_\delta}$, $n K_\sigma \dfrac{L^\delta_\delta}{R_\delta}$, $n K_\pi \dfrac{L^\delta_\delta}{R_\delta}$ and $R_\sigma \dfrac{L^\delta_\delta}{R_\delta}$, very small as compared to the numerical values of the other coefficients figuring in the left members, and render thus also negligible the term $\tau \dfrac{L^\delta_\delta}{R_\delta}$ with respect to the unit. Note that the coefficients $n K^\delta_{ac} \dfrac{L^\delta_p}{R_\delta}$ and $n K^\delta_{bd} \dfrac{L^\delta_s}{R_\delta}$ are large because K^δ_{ac} and K^δ_{bd} are generally very large as compared to $n K_\pi$ and $n K_\sigma$. Under these conditions the roots of the corresponding determinant equated to zero are located on the Gauss plane in the vincinity of the roots of the determinant of the coefficients of following equations:

$$I_{ac}(\tau)\left[R_\pi + \tau\left(L_p + n K^\delta_{ac} \frac{L^\delta_p}{R_\delta}\right)\right] +$$
$$+ I_{bd}(\tau)\left[n K_\sigma + \tau\left(L + n K^\delta_{ac} \frac{L^\delta_s}{R_\delta}\right)\right] =$$
$$\text{(94)} \qquad = n K^\gamma_{ac}\, I_\gamma(\tau)$$
$$I_{ac}(\tau)\left[n K_\pi - \tau\left(L + n K^\delta_{bd} \frac{L^\delta_p}{R_\delta}\right)\right] -$$
$$- I_{bd}(\tau)\left[R_\sigma + \tau\left(L_s + n K^\delta_{bd} \frac{L^\delta_s}{R_\delta}\right)\right] = 0.$$

These equations correspond to same metadyne deprived of the vinculators but with a primary and secondary self inductions, L_p' and L_s', equal to:

$$L_p' = L_p + n K^\delta_{ac} \frac{L^\delta_p}{R_\delta}$$
$$\text{(95)}$$
$$L_s' = L_s + n K^\delta_{bd} \frac{L^\delta_s}{R_\delta}$$

and two mutual inductances, L' and L'' figuring in the first and in the second of equations (94) respectively and having following values:

$$L' = L + n K^\delta_{ac} \frac{L^\delta_s}{R_\delta}$$
$$\text{(96)}$$
$$L'' = L + n K^\delta_{bd} \frac{L^\delta_p}{R_\delta}.$$

We may then derive following deduction:

If the time constant of the circuit of the vinculators is very small as compared to the time constants of the primary and of the secondary circuits, the dynamic behaviour is practically similar to the one of the same machine deprived of the vinculators but endowed with self induction and mutual induction coefficients as indicated by formula (95) and (96).

We may utilize this property in may ways. For instaance we may reduce to zero the value of two of the coefficients, L_p', L_s', L', L'', or reduce them all to small values compared to L_p, L_s, L.

Note that the two vinculator windings traversed by same current may be obtained by only one coil around each polar segment, having a number of turns equal to the algebraic sum of the turns of the two separate coils corresponding to the two separate windings, the sign of the turns changing when the direction of the corresponding ampere turns changes. This arrangement reduces copper losses. If, in particular there is;

$$\left| K^\delta_{ac} \right| = \left| K^\delta_{bd} \right|$$

only two coils per cycle are needed for the simultaneous operation of the two vinculator windings.

Reversely any diagonal vinculator winding closed in itself, i.e. a vinculator winding comprising a single coil around every alternate polar segment, acts as the two vinculator windings 8 and 9 of the scheme of fig. 16.

Such a coil arrangement is often used for reducing copper losses and stator size, this diagonal stator winding being used say as a variator winding energized by an external source. If the external source is then a dynamo or a battery or an alternator, the diagonal variator winding will behave as a diagonal vinculator practically short circuited.

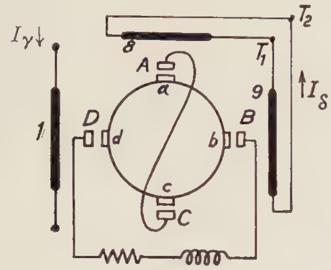


Fig. 16.

Such an action is not desired in the most cases and the arrangement of the diagonal variator winding, must be, generally, renounced at and be replaced by the usual arrangement of two distinct windings, one having its magnetic axis coinciding with the primary commutating axis and the other with the secondary axis correspondingly, each winding being energized by a separate external source.

If the external source can be a metadyne, as fig. 17 shows the undesirable behavior of the diagonal arrangement of the variator winding is substantially reduced when the permanent armature currents of the main

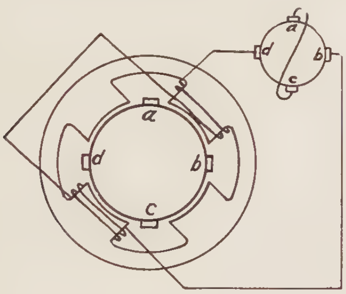


Fig. 17.

metadyne are alternating currents, and it is practically eliminated if the said permanent armature currents are direct currents, as it is demonstrated in section 8 of this chapter. The auxiliary metadyne energizing the diagonal variator winding must have a high value of reactivity resistance, as shown in the above mentioned section. It is obvious that the auxiliary metadyne may energize the variator winding with any kind of current, constant or variable, direct or alternating.

Chapter II.

OUTLINE OF SOME ANALYTICAL METHODS APPLIES IN THIS TEXT

The theory of metadynes was almost completely developed long before any metadyne were constructed and the author disclosed the invention and his work for the first time at the international Montefiore Contest in Liege in 1928.

Having met too laborious calculations in establishing the conditions for a stable operation of the new machines, he devised some methods reducing labor in calculations. Those methods permitted him to check the stability in many cases. They were mentioned in the Montefiore memoire and published in the Revue Generale de l'Electricité, Paris, 1930. They are here a little more developed, so as to authorize the examples contained in this book; they will be reconsidered and more completely handled in a special volume considering stability.

A) *Frontier variety of the stability domain.*

A) The frontier variety of the stability domain.

1.) *Introductory notes.*

The stability problem will be considered more completely in a special volume; here we limit our investigation to some of its aspects, at which we have already referred in the previous chapter, in investigating the nature of the transient currents.

In Chapter I we obtained information about stability by determining the exponent of the exponentials of the solution representing the transient currents; we persist in this chapter in the same method but we choose a different approach.

The dynamic equations considered in Chapter I have following form:

$$\sum_{i=1}^{p} a'_{ik}\, y_i' + \sum_{i=1}^{p} b'_{ik} \frac{dy'_i}{dt} + \ldots + \tag{1}$$

$$\ldots + \sum_{i=1}^{p} m'_{ik} \frac{d y'^{\,m-1}_i}{dt^{m-1}} = A_k$$

$$k = 1,\ 2,\ \ldots,\ p$$

where A_k is a known term, say the perturbance or the driving or exciting function the solutions for a set:

$$y'_i = f\,(a'_{sk},\ b'_{sk},\ \ldots,\ m'_{sk}) \tag{2}$$

$$i = 1,\ 2,\ \ldots,\ p$$

$$s = 1,\ 2,\ \ldots,\ p$$

$$k = 1,\ 2,\ \ldots,\ p$$

of functions of the coefficients figuring in equations (1) and whose expression can be given a form using radicals only when the corresponding characteristic equation is of the fourth degree, and sometimes even when it is of higher degree but of special form.

In the previous chapter we have considered only cases where the characteristic equation, generally derived under the form of a determinant equated to zero, was of low degree.

We know that the solutions are continuous functions of the coefficients and hence that the operation from stable becomes unstable when the coefficients vary continuously and pass through the critical values, as we have many times determined.

If our purpose is to make sure whether the operation is stable or not, and we determine the set of solutions, (2) we are performing an effort harder than it is necessary for our purpose. We may instead adopt following preceeding where we consider an hyperspace having $m\,p^2$ dimensions as many as they are different coef-

ficients in equations (1) in which we separate the domain of stability from the domain of instability.

We may then try to determine the separating hypersurface, obviously constituted by the points corresponding to the critical values of the coefficients. At this hypersurface we will refer as to the « stability frontier ».

If we can establish the stability frontier it will suffice to check whether the point having as p^2 coordinates the actual values of the coefficients, lies in the stability domain or not.

The solution of system (1) may be derived from the solution of a single differential equation, of order, r, where r is generally equal to $p\,(m-1)$. The corresponding characteristic equation will be a polynomial of order r equated to zero:

$$\sum_{i=1}^{r} \alpha_i\, \varrho^i = \alpha_0 \,. \tag{3}$$

The stability depends thus on the sign of the real part of the roots of equation (3).

If then for instance we start from Hurwitz theorem which states that the real part of all said roots is negative if all the coefficients of equation (3) and some determinants constructed with these coefficients have same sign, we may determine the « stability frontiers » by equating to zero said coefficients and determinants; but the mere fact that we must equate to zero all coefficients is discouraging. We choose another approach of the problem by using a preceeding indicated by Cauchy.

2.) *Gauchy's form of a system of differential equations.*

We depart from system (100) and transform it into another equivalent system where only equations of the first degree appear by substituting new variables to represent derivatives of higher order. For instance we replace equation:

$$\sum_{i=1}^{p} d'_{ik} \frac{d^3 y'_i}{dt^3} + \sum_{i=1}^{p} c'_{ik} \frac{d^2 y'_i}{dt^2} + $$

$$+ \sum_{i=1}^{p} b'_{ik} \frac{dy'_i}{dt} + \sum_{i=1}^{p} a'_{ik}\, y'_i = A_k$$

$$k = 1,\ 2,\ \ldots,\ p.$$

by following $2p+1$ equations:

$$\sum_{i=1}^{p} d'_{ik} \frac{dy'_{2p+i}}{dt} + \sum_{i=1}^{p} c'_{ik}\, y'_{2p+i} + \sum_{i=1}^{p} b'_{ik} \frac{dy'_{p+i}}{dt} +$$

$$+ \sum_{i=1}^{p} a'_{ik}\, y'_i = A_k$$

$$y'_{p+h} = \frac{dy_h'}{dt}; \quad y'_{2p+h} = \frac{dy'_{p+h}}{dt}; \quad h = 1,\ 2,\ \ldots,\ p$$

We obtain finally a system of following form:

$$\sum_{i=1}^{r} a_{ik} \frac{dy'_i}{dt} + \sum_{i=1}^{r} b_{ik}\, y_i' = A_k \tag{4}$$

$$k = 1,\ 2,\ \ldots,\ r$$

r being larger than p, and the coefficients a_{ik} and b_{ik} being linear homogeneus combinations of the coefficients of equations (1).

System (4), if irreductible, has an infinity of sets of r, and only r, solutions:

$$y'_{gi} \qquad \begin{matrix} i = 1,\ 2,\ \ldots,\ r \\ g = 1,\ 2,\ \ldots,\ r \end{matrix}$$

linearly independent from one another and each set may be derived from another by linear substitutions. This and other basic theorems mentioned in this chapter will be demonstrated in the volume of « Non linear transients », for the convenience of the reader, but are assumed known here. These sets of solutions form a square matrix of r^2 elements:

$$\| y'_{ik} \| \,.$$

The known members A_k $k = 1, 2, \ldots, r$ do not enter in the research of the exponents of the exponentials of the solutions as we have noticed in all sections of the previous chapter; hence for our particular purpose here, they may be ignored and instead of the system (4) we may investigate following one:

$$\sum_{i=1}^{r} a_{ik} \frac{dy_i}{dt} + \sum_{i=1}^{r} b_{ik}\, y_i = 0 \qquad k = 1,\ 2,\ \ldots,\ r. \tag{5}$$

In « Non linear transients » we shall consider the non homogeneous system (4) in order to determine the transients. The determinant of the coefficients a_{ik} is frequently different from zero, generally, whenever the equations are the dynamic equation of a system involving transfer of energy. The system (5) may be solved for the differentials and take following form

$$\frac{dy_i}{dt} = \sum_{i=1}^{r} c_{ik}\, y_i \qquad k = 1,\ 2,\ \ldots,\ r. \tag{6}$$

Each set of solutions: y_i ($i = 1, 2, \ldots, r$) satisfies system (6) and r linearly independent sets of solutions

$$y_{gi} \qquad \begin{matrix} g = 1,\ 2,\ \ldots,\ r \\ i = 1,\ 2,\ \ldots,\ r \end{matrix}$$

will satisfy following systems:

$$\frac{dy_{gi}}{dt} = \sum_{i=1}^{r} c_{ik}\, y_{gi} \qquad \begin{matrix} k = 1,\ 2,\ \ldots,\ r \\ g = 1,\ 2,\ \ldots,\ r \end{matrix}$$

These systems of equations may be written as a single matrix equation as follows:

$$\left\| \frac{dy_{ik}}{dt} \right\| = \| y_{ik} \| \cdot \| c_{ik} \| \,. \tag{7}$$

We dispose now of the elements for the demonstration of the theorem developed in following section leading to the determination of a part of the frontier variety of the stability domain. We say that equations (6) or (7) have Cauchy's form.

3.) *Theorem on the stability frontier variety without oscillations.*

Let us refer at the frontier variety of the stability domain by symbol $S\,V$. We may distinguish two sections of it, one section, $S\,V\,O$, comprising the critical points corresponding to an operation with oscillating currents, and another section, $S\,V\,N$, constituted by critical points corresponding to unidirectional currents. The following theorem determines the latter.

Theorem. — The section, $S\,V\,N$, of the stability domain frontier of the critical points corresponding to currents tending to infinity without oscillations, is analytically expressed by equating to zero the determinant of the coefficients figuring in Cauchy's form of the homogeneous dynamic equations.

Let us consider Cauchy's form using the matrix symbol (7), and multiply at the right each member by a matrix $\| g_{ik} \|$ having r^2 elements and whose de-

terminant is not zero; we have:

$$(8) \qquad \left\| \frac{dy_{ik}}{dt} \right\| \cdot \| g_{ik} \| = \| y_{ik} \| \cdot \| c_{ik} \| \cdot \| g_{ik} \| .$$

Assuming now that matrix $\| g_{ik} \|$ is composed by constants, and that it has a determinant different from zero, the product:

$$\| y_{ik} \| \cdot \| g_{ik} \|^{-1} = \| y'_{ik} \|$$

is another matrix of r sets of integral solutions linearly independent from one another and constitutes an integral matrix of a system of differential equations, whose matrix of derivatives is obviously:

$$\left\| \frac{d'y_{ik}}{dt} \right\| = \left\| \frac{dy_{ik}}{dt} \right\| \cdot \left\| g_{ik} \right\|^{-1}$$

and we may, therefore, write:

$$\left\| \frac{dy_{ik}}{dt} \right\| \cdot \left\| g_{ik} \right\|^{-1} = \left\| y_{ik} \right\| \cdot \left\| g_{ik} \right\|^{-1} \left\| c_{ik} \right\| .$$

By multiplying at the right both members of the last equation by $\| g_{ik} \|$ we obtain:

$$\left\| \frac{dy_{ik}}{dt} \right\| = \left\| y_{ik} \right\| \cdot \left\| g_{ik} \right\|^{-1} \cdot \left\| c_{ik} \right\| \cdot \left\| g_{ik} \right\|$$

to be compared with equation (107), and which may be written as follows:

$$\left\| \frac{dy_{ik}}{dt} \right\| = \left\| y_{ik} \right\| \cdot \left\| h_{ik} \right\|$$

where matrix $\| h_{ik} \|$ is constituted by constants being linear combinations of the coefficients c_{ik} of our original differential equations. This matrix is referred at as the «*transform matrix of* $\| c_{ik} \|$ by matrix $\| g_{ik} \|$» and its determinant is evidently different from zero. Matrix $\| g_{ik} \|$ is referred at as «transforming matrix»; they are correlated with one another by following relation:

$$(9) \qquad \| h_{ik} \| = \| g_{ik} \|^{-1} \cdot \| c_{ik} \| \cdot \| g_{ik} \| .$$

On the other hand, given a matrix $\| c_{ik} \|$ of constant r^2 elements having its determinant different from zero, we can always determine a transforming matrix, having its determinant different from zero such that the transform takes a very simple form, for instance following form, called «canonical form»:

$$(10) \qquad \left\| \begin{matrix} \varrho_1 & 0 & 0 & \dots & 0 \\ 0 & \varrho_2 & 0 & \dots & 0 \\ 0 & 0 & \varrho_3 & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \varrho_r \end{matrix} \right\| = \left\| \varepsilon_{ik}\, \varrho_k \right\|$$

where the «Kronecker's» symbol ε_{ik} is equal to 1 when $i = k$ and equal to 0 when $i \neq k$. Let $\| d_{ik} \|$ be the transforming matrix. The values $\varrho_1, \varrho_2, \dots \varrho_r$ figuring in (9) are the roots of following equation, referred at as «secular characteristic equation»:

$$(11) \qquad \left| \begin{matrix} c_{11} - \varrho & c_{12} & c_{13} & \dots & c_{1r} \\ c_{21} & c_{22} - \varrho & c_{23} & \dots & c_{2r} \\ \cdot & \cdot & \cdot & \dots & \cdot \\ c_{r1} & c_{r2} & c_{r3} & \dots & c_{rr} - \varrho \end{matrix} \right| = 0$$

This property will be demonstrated in «Non Linear Transients» where matrices are extensively used. The canonical form (10) is obtained only when the secular characteristic equation has not multiple roots. When there are multiple roots the canonical form is less simple; in «Non Linear Transients» multiple roots are also considered, and this demonstration completed, but, for the sake of simplicity, we assume here that (11) has not multiple roots. As a result of the previous transformations we may consider, instead of the original dynamic equations, following one:

$$(12) \qquad \left\| \frac{dy_{ik}}{dt} \right\| = \left\| y_{ik} \right\| \cdot \left\| \varepsilon_{ik}\, \varrho_k \right\| .$$

Yet the integrals of this equation are immediately determined;

$$y_k = A_k\, e^{\varrho_k t} \qquad k = 1, 2, \dots, r$$

A_k being an integration constant. Thus the diagonal terms of matrix (10) are the exponentials of the exponents of the solutions, and there is an integral matrix of the form:

$$\left\| y_{ik} \right\| = \left\| \varepsilon_{ik}\, e^{\varrho_k t} \right\| .$$

Let us decompose the exponentials ϱ_k into real part ϱ_k' and imaginary part ϱ_k'':

$$\varrho_k = \varrho_k' + \varrho''_k$$

and let us vary with continuity the coefficients c_{ik} of the original equations; the components ϱ_k' will vary with continuity and whenever the values of coefficient c_{ik} are such as to equate to zero one of these real components, the corresponding representative point, having as r^2 coordinates said values of c_{ik}, is traversing the stability frontier variety $S\,V$. Thus for every point of $S\,V$ at least one of components ϱ_k', $k = 1, 2, \dots, r$, is becoming zero. But if the representative point is traversing the section $S\,V\,N$ of the frontier variety for which there are no oscillations, then at least one of the diagonal terms of the determinant:

$$\left| \varepsilon_{ik}\, \varrho_k \right|$$

is passing through zero. And reversely; hence necessary and sufficient condition for a representative point to lie on $S\,V\,N$ is to have its r^2 coordinates such as to equante to zero the latter determinant:

$$(13) \qquad \left| \varepsilon_{ik}\, \varrho_k \right| = 0 .$$

Yet there is:

$$\| d_{ik} \|^{-1} \cdot \| c_{ik} \| \cdot \| d_{ik} \| = \| \varepsilon_{ik}\, \varrho_k \| .$$

From this matrix equation we deduce a similar equation for the corresponding determinants:

$$| d_{ik} |^{-1} \cdot | c_{ik} | \cdot | d_{ik} | = | \varepsilon_{ik}\, \varrho_k | .$$

Noting that the transforming matrix $\| d_{ik} \|$ has a determinant different from zero, we deduce that $\| \varepsilon_{ik}\, \varrho_k \|$ is passing through zero when and only when $\| c_{ik} \|$ is passing through zero and the theorem is demonstrated.

4.) *Corollaries.*

Reconsidering the process followed in order to derive the Cauchy's form (7) of the dynamic equation from the original ones, we note that the matrix $\| c_{ik} \|$ is obtained by solving equations (5) for the derivatives $\frac{dy_i}{dt}$ $i = 1, 2, \dots, r$. Using an integral matrix $\| y_{ik} \|$ we may write equations (104) as follows:

$$\left\| \frac{dy_{ik}}{dt} \right\| \cdot \left\| a_{ik} \right\| = - \left\| y_{ik} \right\| \cdot \left\| b_{ik} \right\|$$

and multiplying at right both members by $\| a_{ik} \|^{-1}$, we obtain:

$$(14) \qquad \left\| \frac{dy_{ik}}{dt} \right\| = \| y_{ik} \| \cdot \| b_{ik} \| \cdot \| a_{ik} \|^{-1} = - \| y_{ik} \| \cdot \| c_{ik} \| \cdot \| y_{ik} \| \cdot \| c_{ik} \| .$$

If we assume that the matrix $\| a_{ik} \|$ has a determinant different from zero, the determinant of matrix $\| c_{ik} \|$ is zero only when the determinant of matrix $\| b_{ik} \|$ is zero.

At this point let us subdivide the original dynamic equations into two categories, a first category corresponding to systems where energy variations occur only for the energy stored under one form, say the form due to magnetic field, and a second category corresponding to systems where energy variations occur also for energy stored under many forms, say energy due to electric field, energy due to mechanical movements, energy due to magnetic field.

Let us first consider the first category of equations; then the original dynamic equations are all of the first degree hence already in the form of equations (5) and the coefficients a_{ik} are linear homogeneous combinations of coefficients of, say, self and mutual inductances. In « Non Linear Transients » we consider matrixes constituted by such coefficients and demonstrate that their determinant is different from zero. Thus in the relation already found (see (14)):

$$- \| b_{ik} \| \cdot \| a_{ik} \|^{-1} = \| c_{ik} \|$$

the coefficients b_{ik} correspond to the direct current asymptotic operation, and the determinant of coefficients a_{ik} is not zero; therefore the determinant $| c_{ik} |$ is zero only when the determinant $| b_{ik} |$ is zero, and we may deduce following corollary:

Corollary I. — Section $S\ V\ N$ of the stability domain frontier of the critical points corresponding to variables tending to infinity without oscillations, is analytically expressed by equating to zero the determinant of the coefficients of the static equations corresponding to the asymptotic non oscillating operation, whenever in the dynamic equations figures the variation of only one kind of energy.

In Chapter I, we were careful to apply the investigation of the direct current asymptotic equations only to cases where the dynamic equations registered a variation of energy only due to magnetic field; we indicated there the deductions of the investigation only as probable. Yet a rigurous demonstration based on the arguments there developed would require only a further refinement of same arguments but the latter corollary renders it useless; whatever deduction we derived there as probably becomes now certain.

Let us consider the second category of dynamic equations. We may further subdivide them into two classes, a first class where the derivatives of each variable of highest order in the dynamic equations represent exclusively the variation of only some forms of energy, say forms F 1 and F 2, and a second class where this does not happen.

The process applied to the first class, for deriving the Cauchy's form of equations from the original one leads us to a matrix equation (7) where the coefficients c_{ik} are the coefficient of a system of linear homogenous equations expressing a pseudo static operation where all derivatives representing the variation of forms F 1 and F 2 of energy are missing. We then say that matrix $\| c_{ik} \|$ corresponds to a «pseudo static» operation because a real static operation comprises only the terms of the original dynamic equations without the signal of derivation with respect to time. Now we may apply same arguments as developed for the demonstration of Corollary I, to both classes and assume that the coefficients of the highest order derivatives in each original dynamic equation form a matrix whose determinant is not zero; we then obtain the demonstration of following corollary:

Corollary II. — Section $S\ V\ N$ of the stability domain frontier of the critical points corresponding to variables tending to infinity without oscillations is analytically defined by equating to zero the determinant of the coefficients of the pseudo static equations constituted by the original homogenous dynamic equations deprived of their derivatives of the highest order of each variable and having all other derivatives replaced by new variables.

Corollary III. — If the derivatives of the highest order of each variable in the dynamic equations are the only terms corresponding to a variation of energy of only some forms of energy, say forms F 1 and F 2, the stability frontier variety without oscillations, $S\ V\ N$, is independent from the variations of energy of forms F 1 and F 2, but it depends on the variations of energy of other forms indicated in the dynamic equations.

The main interest of the theorem and its corollaries lies in the fact that they are expressed by equations we may construct ignoring the coefficients of the derivatives of the highest order of each variable.

5.) *Comments and examples.*

The knowledge of the section $S\ V\ N$ of the stability domain frontier is practically more important than the knowledge of the section with oscillations $S\ V\ O$ because the latter is generally preannounced, during the tests by the rising of moderate oscillations.

Under the assumptions made the equation defining variety $S\ V\ N$ may be written under the three following forms:

$$(15) \qquad | c_{ik} | = 0$$

$$(16) \qquad | b_{ik} | = 0$$

$$(17) \qquad | \varepsilon_{ik}\, \varrho_k | = \varrho_1\, \varrho_2 \ldots \varrho_r = 0 \,.$$

We may give to same equation another form by ordinating the secular characteristic equation (11) with respect to ϱ as follows:

$$\begin{vmatrix} c_{11} - \varrho & c_{12} & . & . & c_{1r} \\ c_{21} & c_{22} - \varrho & . & . & c_{2r} \\ c_{r1} & c_{r2} & . & . & c_{rr} - \varrho \end{vmatrix} = c_r\, \varrho^r + c_{r-1}\, \varrho^{r-1} + \ldots + c_1\, \varrho + C_0 = 0$$

and remembering that the product of the roots of said equation is equal to $(-1)^r\, c_0$; we may write hence as equation of $S\ V\ N$ following one:

$$(18) \qquad c_0 = 0 .$$

Equations (15), (16), (17) and (18) are of order r with respect to the coefficients figuring either in matrix $\| c_{ik} \|$ or in matrix $\| b_{ik} \|$.

In a given problem the variables are constructional or operational quantities, figuring in the algebraic expressions of the above indicated coefficients. Such quantities are the number of turns of some stator windings, the angular location of their magnetic axis with respect to same brushes, the rotational speed of the armature, and so on. Let us call $x_1, x_2, \ldots x_g$ these quantities; equations (15), (16), (17), ((18) yield as many equations in in $x_1, x_2, \ldots x_g$ which generally are of an order, say q, different from r, and frequently $q < r$.

The hyperspace, in which we are then interested, has only q dimensions; if $q = 3$, we have an three de-

mensional space, if $q = 2$, a surface and if $q = 1$, just a line, and variety $S\,V\,N$ is then reduced to a surface, to a line and to a set of points, respectively. In all examples considered in Chapter I, we had only variable parameter, generally the rotational speed, n.

Let us now give some examples of application.

A first one may be the case of two shunt windings connected between the secondary brushes treated in section 10. The dynamic equations are all of the first degree and the differential terms correspond to variation of energy of the magnetic field form; thus Corollary I applies and the determinant of the coefficients of the static equations, equated to zero represents the section $S\,V\,N$.

Fig. 18 shows the scheme of a second example corresponding to the two last corollaries. The homogeneous dynamic equations are, with the usual symbols:

$$(19)\qquad \begin{aligned} &\left[L_p \frac{d^2}{dt^2} + R_\pi \frac{d}{dt} + S_p\right] I_{ac} + \left[n K_\sigma \frac{d}{dt} + L \frac{d^2}{dt^2}\right] I_{bd} = 0 \\ &\left[-L \frac{d^2}{dt^2} + n K_\pi \frac{d}{dt}\right] I_{ac} - \left[L_s \frac{d^2}{dt^2} + R_\sigma \frac{d}{dt} + S_s\right] I_{bd} = 0. \end{aligned}$$

In order to give the Cauchy's form to these equations we put

$$(20)\qquad y_3 = \frac{d}{dt} I_{ac} \quad ; \quad y_4 = \frac{d}{dt} I_{bd}$$

and we substitute in equations (19). The latter thus transformed added to equations (20) form a set of equations whose matrix of the coefficients of the non differential terms is:

$$\begin{Vmatrix} S_p & 0 & R_\pi & n K_\sigma \\ 0 & S_s & n K_\pi & R_\sigma \\ 0 & 0 & 1 & 0 \\ & 0 & 0 & 1 \end{Vmatrix}$$

the first, second, third and fourth colums corresponding to variables I_{ac}, I_{bd}, y_3 and y_4 respectively, and the first, second, third and fourth row corresponding to

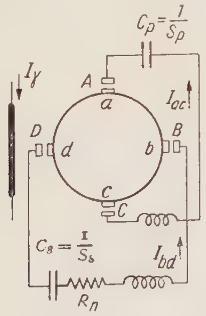


Fig. 18.

the first, the second, of equations (19), the first and the second of equations (20) respectively.

The determinant of this matrix is: $S_p S_s$ which, equated to zero, show that the stability frontier variety without oscillations is defined by capacitors of infinite capacity and that as long as the capacitors will have a finite value the stability frontier is completely constituted by points corresponding to oscillations.

For a third example we refer to fig. 19 showing the scheme of an S generator metadyne provided with series winding suggested by rectangles A, B, C, D, opposite to brushes a, b, c, d, respectively, further comprising a secondary variator winding 1 and a primary vinculator 8, traversed by current I_δ and connected to capacitor $C_\varepsilon = \frac{1}{S_\varepsilon}$, in parallel with an inductance, L_ε having a resistance R_ε.

The homogeneous differential equations are:

$$\left(R_\pi + L_p \frac{d}{dt}\right) I_{ac} + \left(n K_\sigma + L \frac{d}{dt}\right) I_{bd} + $$
$$+ L_p^\delta \frac{d}{dt} I_\delta = 0$$

$$\left(n K_\pi - L \frac{d}{dt}\right) I_{ac} - \left(R_\sigma + L_s \frac{d}{dt}\right) I_{bd} +$$
$$+ n K_{bd}^\delta I_\delta = 0$$

$$(21)\qquad - L_p^\delta \frac{d^2}{dt^2} I_{ac} - \left(R_\delta \frac{d}{dt} + L_\delta \frac{d^2}{dt^2}\right) I_\delta -$$
$$- S_\varepsilon I_\varepsilon = 0$$

$$- S_\varepsilon I_\varepsilon - L_\varepsilon \frac{d^2}{dt^2} I_\varepsilon + L_\varepsilon \frac{d^2}{dt^2} I_\delta -$$
$$- R_\varepsilon \frac{d}{dt} (I_\varepsilon - I_\delta) = 0.$$

Let us add to this system following equations with new variables:

$$(22)\qquad \frac{d}{dt} I_{ac} = y_5 \quad ; \quad \frac{dI_\delta}{dt} = y_6 \quad ; \quad \frac{dI_\varepsilon}{dt} = y_7 .$$

Substituting in equations (21) and ordering, we obtain:

$$+ L \frac{d}{dt} I_{bd} + R_\pi I_{ac} + n K_\sigma I_{bd} + L_p y_5 +$$
$$+ L_p^\delta y_6 = 0$$

$$- L_s \frac{d}{dt} I_{bd} + n K_\pi I_{ac} - R_\sigma I_{bd} +$$
$$+ n K_{bd}^\delta I_\delta - L y_5 = 0$$

$$(23)\qquad - R_\delta \frac{d}{dt} I_\delta - L_p^\delta \frac{d}{dt} y_5 - L_\delta \frac{d}{dt} y_6 - S_\varepsilon I_\varepsilon = 0$$

$$+ L_\varepsilon \frac{d}{dt} y_6 - L_\varepsilon \frac{d}{dt} y_7 + R_\varepsilon y_6 - R_\varepsilon y_7 -$$
$$- S_\varepsilon I_\varepsilon = 0.$$

A simple inspection of the two first equations (23) shows that the the determinant of the coefficients of the derivatives is zero, hence there is a linear relation

between the variables. We obtain this relation by eliminating $\frac{d}{dt} I_{bd}$ between said equations:

$$
\begin{aligned}
& I_{ac}(R_\pi L_s + n K_\pi L) + I_{bd}(n K_\sigma L_s - R_s L) + \\
(24) \qquad & + I_\delta n K^\delta_{bd} L + y_5 (L_p L_s - L^2) + \\
& + L_s L^\delta_p y_6 = 0
\end{aligned}
$$

by means of which we may express one of the variables, say I_δ, as a linear homogeneous function of the others:

$$I_\delta = A\, I_{ac} + B\, I_{bd} + (C y_5 + D y_6)$$

the coefficients A, B, C and D being readily determined from (24). Substituting in the three last equations (23) and in the (22) we reduce the system to six equations and the matrix of the coefficients of the variables to 6×6 elements. Equating to zero the corresponding determinant we obtain the equation determining the variety $S\,V\,N$. In this determinant figure coefficients corresponding to all forms of energy considered in this case.

B) Approach through a simplified characteristic equation.

6.) *Outline of the method.*

In Chapter I we have investigated some cases where the characteristic equation is of a degree higher than two, by observing that some terms of the equation could be neglected for approximate results, and by solving the simplified equation. We reconsider here this proceeding and develop it.

Inspecting an algebraic equation we may judge that the omission of some term would not modify basically the location of the roots on the Gauss plane; well known rules for numerical approximate solution apply same fundamental concept. Having located on the Gauss plane the roots of the simplified equation assumed solved, we reconsider the original equation comprising all terms and either we determine the approximate numerical values of its roots, or we try to circumscribe an area of the Gauss plane in which said roots lie, reducing said area as much as possible. The location of this area in the Gauss plane informs us about the nature of the roots and hence about the mode of operation. In fact the most important information is the one regarding stability and quick response.

We remind the most commonforms of algebraic equations permitting a ready expression of their roots as algebraic functions of the coefficients.

$$
(25) \qquad
\begin{aligned}
& a x^2 + b x + c = 0; && (a x^2 + b x + c)^q = d \\
& a x^3 + b x + c = 0; && (a x^3 + b x + c)^q = d \\
& a x^{2p} + b x^p + c = 0; && (a x^{2p} + b x^p + c)^q = d \\
& a x^{3p} + b x^p + c = 0; && (a x^{3p} + b x^p + c)^q = d
\end{aligned}
$$

By conveniently modifying the variable parameters we dispose, we may shift the roots of the simplified equation well within the portion of the Gauss plane we want to have the roots. Then around each of said roots we draw closed integral lines, γ, along which we calculate Cauchy's integral:

$$
(26) \qquad \int_\gamma \frac{\frac{df(z)}{dz}}{f(z)}\, dz = 2\,\pi\, j\,(M - N)
$$

where $f(z)$ is a meromorph analytical function of the complex variables within the area comprised by line γ, and an holomorph, different from zero, one upon said line γ, and where M and N are respectively the number of the zero points and of the pole points of function $f(z)$ lying within said area, each zero point and pole point being reckoned as many times as there are units in their order of multiplicity.

The function $f(z)$ we consider here is the polynomial corresponding to the original characteristic equation and therefore M is the number of its roots taking into account their order of multiplicity and N is zero because the only poles of our function lie at the infinite, while we will keep the integral line γ at finite distance from the origin.

We proceed calculating such integrals until we find where all roots are located and until a sufficient degree of approximation of the location of the roots is reached.

During calculation of the Cauchy integral we must check, that the function under the integral be holomorph along the integral line; this is easy because if $f(z)$ is holomorph, its derivative is also holomorph and if $f(z)$ is diminishing towards zero, we have practically located a root, and either we have found all roots and we may discontinue the calculation of the Cauchy's integral or we increase the area comprised by γ in order to include all roots sought.

If the simplified characteristic equation is derived from the original one by elimination of the highest powers of the polynomial, the complementary roots of the original characteristic equation spring out from the roots of the simplified equation (see fig. 20, where the roots of the simplified equation, of the 3rd order, are three, a, b, c, and the roots of the original equation are six: a' a'', b' b'', c' c''). If the simplified equation is derived by eliminating the lowest order terms, the complementary roots spring out from the origin of the Gauss plane. The method of using the simplified characteristic equation and solving it, has upon the method of approximate numerical solution of the original characteristic equation, the advantage to permit a convenient choice of the variable parameters in order to bring the roots of the simplified characteristic equation, well within the desired area of the Gauss plane.

7). *Example.*

Let us reconsider again the S generator metadyne of fig. 19 and start from dynamical equations (21), where, for the sake of simplicity the coefficient of the mutual

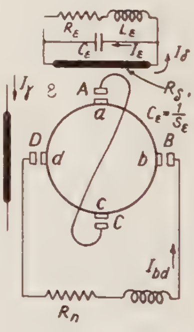


Fig. 19.

induction L, is assumed being equal to zero, set to this value in any of the ways indicated in the previous chapter.

The corresponding Laplace transformed equations

are:

$$(R_\pi + \tau L_p)\, I_{ac}(\tau) + n K_\sigma I_{bd}(\tau) + $$
$$+ L_p^\delta \tau I_\delta(\tau) = n K_{ac}^\gamma I_\gamma(\tau)$$
$$n K_\pi I_{ac}(\tau) - (R_\sigma + \tau L_s)\, I_{bd}(\tau) + $$
$$+ n K_{bd}^\delta I_\delta(\tau) = 0$$
$$-\tau^2 L_p^\delta I_{ac}(\tau) - (\tau R_\delta + \tau^2 L_\delta)\, I_\delta(\tau) -$$
(27)
$$- S_\varepsilon I_\varepsilon(\tau) = - L_p^\delta \frac{d}{dt} I_{ac}(t = 0) -$$
$$- L_\delta \frac{d}{dt} I_\delta(t = 0)$$
$$-[S_\varepsilon + \tau R_\varepsilon + \tau^2 L_\varepsilon]\, I_\varepsilon(\tau) +$$
$$+ [\tau^2 L_\varepsilon + \tau R_\varepsilon]\, I_\varepsilon(\tau) =$$
$$= - L_\varepsilon \left(\frac{d}{dt} I_\varepsilon(t = 0) \frac{d}{dt} I_\delta(t = 0) \right)$$

where we have assumed that the exciting function $I_\gamma(t)$ may be chosen at will, and that the original values of all currents is zero.

We may eliminate $I_\varepsilon(r)$ from the two last equations. We replace thus the third equation of system (27) by following one:

$$-\tau^2 L_p^\delta (\tau^2 L_\varepsilon + \tau R_\varepsilon + S_\varepsilon)\, I_{ac}(\tau) -$$
$$-[(\tau^2 L_\delta + \tau R_\delta)(\tau^2 L_\varepsilon + \tau R_\varepsilon + S_\varepsilon) +$$
$$+ S_\varepsilon(\tau^2 L_\varepsilon + \tau R_\varepsilon)]\, I_\delta(\tau) =$$
(28)
$$= S_\varepsilon L_\varepsilon \frac{d}{dt} I_\varepsilon(t = 0) -$$
$$-[S_\varepsilon L_\varepsilon + L_\delta(\tau^2 L_\varepsilon + \tau R_\varepsilon + S_\varepsilon)]$$
$$\frac{d}{dt} I_\delta(t = 0).$$

In this way we may investigate the system of three equations composed with three unknowns. The determinant of the coefficients of the corresponding homogeneous system equated to zero gives:

(29)
$$\begin{vmatrix} R_\pi + \tau L_p & n K_\sigma & \tau L_p^\delta \\ n K_\pi & -(R_\sigma + \tau L_s) & n K_{bd}^\delta \\ -\tau^2 L_p^\delta(\tau^2 L_\varepsilon + \tau R_\varepsilon + S_\varepsilon) & 0 & -[(\tau^2 L_\delta + \tau R_\delta + S_\varepsilon)(\tau^2 L_\varepsilon + \tau R_\varepsilon + S_\varepsilon) - (S_\varepsilon)^2] \end{vmatrix} = 0.$$

We can set the machine so that $R_\pi = R_\sigma = 0$ as we have seen in Chapter I; we may also set $R_\varepsilon = R_\delta$ as ve will see later on but for the time being we will neglect the terms expressed by R_ε or R_δ and thus obtain instead of the above original equation following simplified one:

$$\tau^6\, L_s L_\varepsilon [L_p L_\delta - (L_p^\delta)^2] + \tau^4 [n^2 X_\sigma K_\pi L_\delta L_\varepsilon +$$
$$+ S_\varepsilon L_p L_s (L_\delta + L_\varepsilon) - n^2 K_\sigma K_{bd}^\delta$$
$$L_\varepsilon L_p^\delta - S_\varepsilon L_s (L_p^\delta)^2] + \tau^2 n^2 K_\sigma S_\varepsilon [K_\pi (L_\delta + L_\varepsilon) -$$
$$- K_{bd}^\delta L_p^\delta] = a\tau^6 + b\tau^4 + c\tau^2 = 0.$$

This equation has two roots equal to zero and the other four may be expressed by following relation:

$$\tau^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\tau_1{}^2,\ \tau_2{}^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Let us consider the sign of therms a, b, and c. The latter is obviously positive. Note that $K_\pi L_\delta$ is equal to $K_{bd}^\delta L_p^\delta$ and hence b and c simplify as follows:

$$b = + S_\varepsilon L_s [L_p L_\delta - (L_p^\delta)^2 + L_p L_\varepsilon]$$
$$c = n^2 K_\sigma K_\pi L_\varepsilon S_\varepsilon$$

both essentially positive.

Thus the two values of $\tau_1{}^2$, $\tau_2{}^2$ will be negative whenever following relation is satisfied:

(30)
$$4[L_p L_\delta - (L_p^\delta)^2]\, n^2 K_\sigma K_\pi L_\varepsilon^2 <$$
$$< S_\varepsilon L_s [L_p L_\delta - (L_p^\delta)^2 + L_p L_\varepsilon]^2.$$

In this relation we may consider as practically easily variable parameters, following quantites: S_ε, n and L_ε; they may be set without -changing windings of the machine.

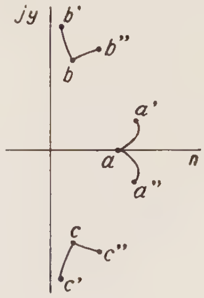


Fig. 20.

When (30) is satisfied the factors composing the polynomial of the reduced equation are:

$$(\tau - 0)^2 (\tau^2 - \tau_1{}^2)(\tau^2 - \tau_2{}^2)$$

and there will be two steady sinusoidal currents having as pulsation $\sqrt{-\tau_1{}^2}$ and $\sqrt{-\tau_2{}^2}$.

If the two members of relation (30) become equal,

(31)
$$4[L_p L_\delta - (L_p^\delta)^2]\, n^2 K_\sigma K_\pi L_\varepsilon =$$
$$= S_\varepsilon L_s [L_p L_\delta - (L_p^\delta)^2 + L_p L_\varepsilon]^2$$

the square, say $\tau_2{}^2$, becomes zero and there will be only one sinusoidal current of pulsation $\sqrt{-\tau_2{}^2}$ and the factors of the polynomial of the reduced equation are

$$(\tau - 0)^4 (\tau^2 - \tau_2{}^2).$$

For satisfying equation (31) and simultaneously giving to pulsation $\sqrt{-\tau^2}$ a predetermined value, we dispose of three easly set parameters, L_ε, S_ε and n.

Let us assume that the two parameters L_ε and S_ε are set such as to satisfy equation (31) with a predetermined value of the speed n; we may now investigate how the roots of the original characteristic equation vary when resistances R_δ and R_ε departing from zero vary and reach values in agreement with the practical construction of the machine.

We return to the original characteristic equation (27) (simplified by the assumption that $R_\pi = R_\sigma = 0$ of it is so desired) which we may write as follow:

$$f(\tau) = 0$$

where $f(\tau)$ is a polynomial of the sixth degree in τ, and apply Cauchy's integral formula (26).

The roots of the simplified characteristic equation are located on the Gauss plane on the imaginary axis, four on the origin and two at points $j\sqrt{-\tau_2^2}$ and $-j\sqrt{-\tau_2^2}$ as indicated on fig. 21. Four roots will spring from the origin say to points $a\ b\ c\ d$ and two will spring from points $\pm j\sqrt{-\tau^2}$ say to points g and h. As integral lines we may choose the two rectangles indicated by dotted lines and for convenience and labor saving we may record graphically the operation step by step by taking as variable the length of the integral line.

If after some repetitions of the integration we have approximately located the roots as fig. 21 shows, we may deduce that there will be six transient terms corresponding to points, a, b, c, d tending to zero and two transient terms corresponding to points g and h tending to infinite through oscillation. It is very probable that the latter will stabilize on a single slightly distorted

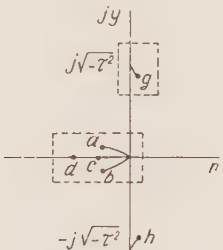


Fig. 21.

sinusoidal, because of the iron saturation. This hypothesis may be checked by changing the coefficients according to saturated iron and finding that then the roots g and h tend to enter into the left side semiplane.

If the machine is available it will generally suffice to determine the roots of the simplified characteristic and to proceed by test to a final setting.

C) Proceeding based on the « practical duration » of transients.

8) *Preliminary definitions.*

Let us consider a variable, quantity, $x(t)$, uniform function of time, t, when the independent variable, t, tends to infinite. It is frequently possible to determine a function $y(t)$, having, generally, a simple expression, such as to have:

$$\lim_{t \to \infty} \left| y(t) - x(t) \right| = 0. \tag{32}$$

We say then that $x(t)$ tends asymptotically towards $y(t)$ and we refer at the latter as the « asymptotic » function of $x(t)$.

We call the difference: $z(t) = y(t) - x(t)$ the « vanishing » transient of $x(t)$ with respect to its asymptotic function $y(t)$.

We may construct many asymptotic functions $y_1(t)$, $y_2(t)$, . . . , $y_h(t)$; there will be then as many « vanishing » transients. We will henceforth assume that we have chosen only one asymptotic function, the most adequate for the special purpose set.

For approximate calculation involving the function $x(t)$ after the instant t_1, the asymptotic function $y(t)$ may substitute $x(t)$, provided the vanishing transient $z(t)$ has for $t > t_1$ a modulus smaller than a predetermined value ε.

Assuming that $x(t)$ is a heavisidean function beginning at $t = 0$, we say that the practical duration of the vanishing transient $z(t)$ is t_{pd} if

$$|z(h)| < \varepsilon \text{ for } t > t_{pd}.$$

The value of ε is arbitrarily chosen according to the degree of exactitude desired.

When many variables x_a, x_b, . . . , x_m figure in a system of differential equations, it is frequently possible to foresee the nature and sometimes even the values of the corresponding asymptotic function and the relative vanishing transients.

In this case we frequently find that the practical duration of the vanishing transients are very different from one another, say ten, hunderd, thousand times larger; we may then subdivide the variables into many groups say the group G_1 comprising variariables x_a, x_b, . . . x_g, group G_2 comprising variables x_{g+1}, x_{g+2}, . . . x_h and group G_3 comprising variables x_{h+1}, x_{h+2}, . . . x_m, the practical duration of the vanishing transients of the first group being, say, of the order of $1''/1000$, the order of the practical duration of the second group being, say, of $1''/10$ and the order corresponding to the third group being, say, of $2''$.

We further, knowing the nature of the solutions, may foresee that the derivatives $\frac{dx_h(t)}{dt}$, $h = a, b, \ldots$ . . . m are small and to zero for any instant having a value larger than the practical duration.

We may derive informations abouth the asymptotic values by investigating the static equations of the system taking into account the results of the application of the methods described in the previous sections of this chapter.

Under these conditions a drastic simplification of the approximate solution of the dynamic equation may be obtained by applying the proceeding given in the following section.

9.) *Description of the proceeding based on the practical duration of the vanishing transients.*

Let us consider the system, S, of m differential equations in the m variables $x_a(t)$, $x_b(t)$, . . . , $x_m(t)$, describing the dynamic bahavior of a transient operation, all said variables being heavisidean beginning at $t = 0$, and where we are able to subdivide the variables into three groups G_1, G_2 and G_3 having a practical duration of their corresponding vanishing transients of different order when passing from one group to another. Let be $x_a(t)$, $x_b(t)$, . . . , $x_g(t)$, the variables of group G_1 and t_{pd} the approximate value of their practical duration; $x_{g+1}(t)$, $x_{g+2}(t)$, . . . , $x_m(t)$ and t'_{pd} the corresponding quantities to group G_2; $x_{h+1}(t)$, $x_{h+2}(t)$, ... , $x_m(t)$ and t''_{pd} the ones corresponding to Group G_3, where there is:

$$t_{pd} < t'_{pd} < t''_{pd}. \tag{33}$$

In order to check this fact, we may consider the dynamic behavior of the variables of one group while the variables of all other groups are kept constant, and preferably different from zero.

We assume that the investigation of the static equations has yielded the asymptotic values, $X_a, X_b, \ldots, X_m$ of the variables.

In the system S we delete the terms with the derivatives of variables $x_{g+1}(t), \ldots, x_{h+1}(t), \ldots, x_m(t)$ as factors, and we substitute the arbitrary, non zero values $A_{g+1}, A_{g+2}, \ldots, A_{h+1}, \ldots, A_m$, to the variables $x_{g+1}(t), \ldots, x_m(t)$ in all other terms where these variables are figuring and we obtain a system, S_1 where the unknowns are only the variables $x_a(t)$, $x_b(t), \ldots, x_g(t)$. Within this system we choose a system, S_g, of only g equations independent from one another and able to yield the solutions for the last mentioned variables. These solutions will thus be expressed as functions of the arbitrary values $A_{g+1}, \ldots, A_m$.

We return to system S and delete all terms having a derivative of functions $x_{h+1}(t), x_{h+2}(t), \ldots, x_m(t)$; we substitute the non zero values $A_{h+1}, A_{h+2}, \ldots, A_m$ to said variables in all terms where they appear and further we substitute the asymptotic values $X_a, X_b, \ldots, X_g$ to $x_a(t), x_b(t), \ldots, x_g(t)$ and in case the asymptotic values of these variables are periodic functions, we substitute their derivatives to the corresponding derivatives of said variables. We obtain thus a system S_2 where the unknowns are only the variables $x_{g+1}(t)$, $x_{g+2}(t), \ldots, x_h(t)$. Within this system we choose a system S_h of only h-g equations independent from one another and able to yield the solution for these variables, solutions expressed in function of $X_a, X_b, \ldots, X_g$ and of $A_{h+1}, A_{h+2}, \ldots, A_m$.

Again we turn to system S and substitute the asymptotic values $X_a, X_b, \ldots, X_{g+1}, \ldots, X_h$ to the corresponding variables and in case these values are periodic, we substitute their derivatives to the derivatives of the corresponding variables. We obtain thus a system, S_3, where the unknowns are only the variables $x_{h+1}(t)$, $x_{h+2}(t), \ldots, x_m(t)$. Within this system we choose a system, S_m, of only $m - h$ equations independent from one another and able to yield the solutions for the last variables, solutions expressed in function of X_a, $X_b, \ldots, X_h$.

At this point we check whether the assumptions upon which the previous operations were based are correct, and, if necessary, we may repeat the operations modifying conveniently the assumptions. If we arrive to satisfactory results, then from the value that the variables $x_{h+1}(t), x_{h+2}(t), \ldots, x_m(t)$, take at an instant, t_1, we readily derive the corresponding values taken by the variable $x_{g+1}(t), x_{g+2}(t), \ldots, x_h(t)$, and finally we derive the corresponding values taken by the variables $x_a(t)$, $x_b(t), \ldots, x_g(t)$.

This proceeding, when it succeeds, splits the given system S of m differential equations, into three separate systems S_g, S_h and S_m of g, h-g, and m-h differential equations.

Whe have considered three groups, G_1, G_2 and G_3; similar proceeding may be applied to any number of groups.

If in particular we can consider as many groups as there are equations, the solution of m simultaneous differential equations is reduced to the solution of m independent single operations.

The proceeding yields only approximate solutions and the approximation is higher the more emphatic is the relation (33).

Chapter III

METADYNE GENERATORS, MOTORS AND TRANSFORMERS

A) *Metadyne Generators.*

1.) C generator metadyne.
2.) F generator metadyne.
3.) D Generator metadyne.
4.) Alpha motor metadyne.
5.) Delta motor metadyne.
6.) Theta and Gamma motors.
7.) Metadyne transformers; the « Cross Transformer ».
8.) The Caduceus transformer.
9.) The Saturn transformer.
10.) Some general remarks.

A) Metadyne Generators.

1.) *C generator metadyne.*

Fig. 22 shows its scheme: four equidistant brushes per cycle, a, b, c, d, the armature being driven at a generally constant speed, n, by a prime mover. Between brushes and corresponding terminals, A, B, C and D any series winding may be inserted having as magnetic axis the commutating axis of the primary brushes, a, c, or that of the secondary brushes, b, d, The primary terminals A and C are connected to a constant voltage, V, line; the secondary terminals, B and D, are shown connected to an external circuit having a resistance,

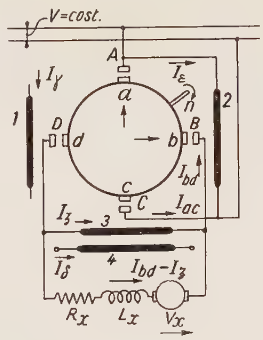


Fig. 22.

R_x, a reactance L_x, and inducing an electromotive force V_x. There are further following stator windings: a secondary variator winding 1, traversed by I_γ, a secondary shunt winding, 2, connected across the primary terminals, and traversed by I_ε; a primary variator winding, 4, traversed by I_δ, and a primary shunt winding, 3, traversed by I_ζ. The arrows indicate the direction of the ampere turns for a positive value of the current.

The normal operation current, may be direct current or alternating current; the nature of the current does not modify substantially our dynamic equations and the final results here under obtained; of course if alternating current is considered, phase transformers will be generally adopted not represented in the figure but readily determined by following the example investigated in « Metadyne Periodics ».

Let us continue to adopt the usual symbols and write the Laplace transformed equations:

$$
\begin{aligned}
&(R_\pi + \tau L_p)\, I_{ac}(\tau) + (n K_\sigma + \tau L)\, I_{bd}(\tau) - \\
(1) \qquad &- \tau L_{ac}^{\delta} I_\delta(\tau) - \tau L_{ac}^{\zeta} I_\zeta(\tau) = \\
&= V(\tau) + n K_{ac}^{\gamma} I_\gamma(\tau) - n K_{ac}^{\varepsilon} I_\varepsilon(\tau)
\end{aligned}
$$

$$(n\,K_\pi - \tau\,L)\,I_{ac}(\tau) - (R_\sigma + \tau\,L_s)\,I_{bd}(\tau) -$$

(2) $$- \tau\,L^{\gamma}_{bd}\,I_\gamma(\tau) - \tau\,L^{\varepsilon}_{bd}\,I_\varepsilon(\tau) =$$

$$= - V_x(\tau) - n\,K^{\delta}_{bd}\,I_\delta(\tau) - n\,K^{\zeta}_{bd}\,I_\zeta(\tau)$$

$$V(\tau) - (R_\varepsilon + \tau\,L_\varepsilon)\,I_\varepsilon(\tau) - \tau\,L^{bd}_{\varepsilon}\,I_{bd}(\tau) -$$

(3) $$- \tau\,L^{\gamma}_{\varepsilon}\,I_\gamma(\tau) = 0$$

$$- (R_x + \tau\,L_x)\,(I_{bd}(\tau) - I_\zeta(\tau)) + V_x(\tau) +$$

(4) $$+ [R_\zeta - R_x + \tau\,(L_\zeta - L_x)]\,I_\zeta(\tau)$$

$$- \tau\,L^{ab}_{\zeta}\,I_{ab}(\tau) - \tau\,L^{\delta}_{\zeta}\,I_\delta(\tau) = 0.$$

The symbol R_σ is the sum of all coefficients that multiplying the current I_{bd} will give the e.m.f. induced in the secondary circuit being proportional to current I_{bd}, which comprise a possible component e.m.f. of V_x, component due to a series field winding of the rotating machine M.

Voltages V and V_x may be unidirectional or alternating; similarly I_γ and I_δ may be unidirectional or alternating.

We may assume that all currents and voltages are heavisidean beginning at $t = 0$. Then the nature of the vanishing transient is determined by the solution of the above written dynamic equations after deleting following terms:

$V(\tau)$, $V_x(\tau)$, $I_\gamma(\tau)$, $I_\delta(\tau)$; let us refer to the thus obtained system of four equation by the letter S. There will be four unknown variables in S: $I_{ac}(\tau)$, $I_{bd}(\tau)$, $I_\varepsilon(\tau)$ and $I_\zeta(\tau)$. Our interest will thus be focused on the determinant of the coefficients of the four last mentioned variables.

Calculation of the roots and their comments will be laborious.

equations as follows:

$$(R_\pi + \tau\,L_p)\,I_{ac}(\tau) + (n\,K_\sigma + \tau\,L)\,I_{bd}(\tau) =$$

$$= \left(1 - n\,K^{\varepsilon}_{ac}\,\frac{1}{R_\varepsilon}\right) V(\tau) + n\,K^{\gamma}_{ac}\,I_\gamma(\tau)$$

(5) $$n\,(K_\pi - \tau\,L)\,I_{ac}(\tau) -$$

$$- \left[R_\sigma + \tau\,L_s - n\,K^{\zeta}_{bd}\,\frac{1}{R\zeta}\,(R_x + \tau\,L_x)\right] I_{bd}(\tau) =$$

$$- V_x(\tau)\left[1 - n\,K^{\zeta}_{bd}\,\frac{1}{R\zeta}\right].$$

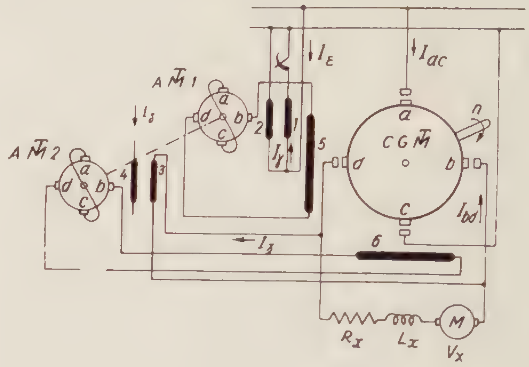


Fig. 23.

The equations are now only two as equations (3) and (4) of the original system are, in a simple way, incorporated into equations (5).

The determinant of the coefficients of the unknowns, $I_{ac}(\tau)$ and $I_{bd}(\tau)$, is a simple one and the corresponding characteristic equation is:

(6) $$\begin{vmatrix} R_\pi + \tau\,L_p & n\,K_\sigma + \tau\,L \\ n\,K_\pi - \tau\,L & - \left[R_\sigma + \tau\,L_s - n\,K^{\zeta}_{bd}\,\dfrac{1}{R\zeta}\,(R_x + \tau\,L_x)\right] \end{vmatrix} = 0$$

We take advantage of the results obtained in Chapter I for deriving a solution which is approximate but easily

a second order equation of the form: $a\,\tau^2 + b\,\tau + c = 0$ where

$$a = L_p\left(L_s - n\,K^{\zeta}_{bd}\,\frac{L_x}{R\zeta}\right) - L^2$$

(7) $$b = L_p\left(R_\sigma - n\,K^{\zeta}_{bd}\,\frac{R_x}{R\zeta}\right) + R_\pi\left(L_s - n\,K^{\zeta}_{bd}\,\frac{L_x}{R\zeta}\right) + n\,L\,(K_\pi - K_\sigma)$$

$$c = R_\pi\left(R_\sigma - n\,K^{\zeta}_{bd}\,\frac{R_x}{R\zeta}\right) + n^2\,K_\sigma\,K_\pi\,.$$

and quickly obtained. For this purpose we adopt the arrangement shown by fig. 23 using two amplifier metadynes $A\,M_1$ and $A\,M_2$ by the intermediation of which variator currents I_γ and I_δ and the shunt winding currents I_ε and I_ζ are acting upon the C.G.M.

The power absorbed by the secondary variator windings of the amplifier metadyne is so small that we can easily afford to insert in their circuit a large resistance. Under these conditions and taking into account the results of section 7 of Chapter I, we may admit with a fair approximation that the following mutual inductances: L^{γ}_{bd}, L^{ε}_{γ}, L^{ε}_{bd}, L^{δ}_{ac}, L^{ζ}_{δ}, L^{ζ}_{ac} are practically zero and simplify the system of the Laplace transformed

The nature of the roots of equation (6) is readily recognized and the relative comments are easily drawn from values (7). For a simple example let us assume that the metadyne is planned in such a way that the mutual induction, L, between primary and secondary circuit is zero; let us further assume that winding 3 is arranged so as to fulfill following relation:

(8) $$R\zeta\,L_s - n\,K^{\zeta}_{bd}\,L_x = 0$$

Under these conditions equation (6) is reduced to a first degree one whose root is:

$$\varrho = \frac{R_\pi\,(R_\sigma\,R\zeta - n\,K^{\zeta}_{bd}\,R_x) + R\zeta\,n^2\,K_\pi\,K_\sigma}{L_p\,(R_\sigma\,R\zeta - n\,K^{\zeta}_{bd}\,R_x)} =$$

and the system comprising the main and auxiliaries metadynes and the external circuit has a dynamic behavior practically equivalent to a simple circuit having a time constant equal to:

$$T = \frac{L_p}{R_\pi} \cdot \frac{1}{1 + \dfrac{n^2 K_\pi K_\sigma}{R_\pi R_\sigma - n K^{\zeta}_{bd} \dfrac{R_x R_\pi}{R_\zeta}}} \tag{9}$$

if there is

$$R_\sigma R_\zeta - n K^{\zeta}_{bd} R_x \neq 0$$

(10) and $$K_\pi K_\sigma = 0.$$

Then

$$T = \frac{L_p}{R_\pi}.$$

If there is

(11) $$R_\sigma R_\zeta - n K^{\zeta}_{bd} R_x > 0$$

and $$K_\pi K_\sigma > 0$$

or inversely:

(12) $$R_\sigma R_\zeta - n K^{\zeta}_{bd} R_x < 0$$

and $$K_\pi K_\sigma < 0$$

the time constant T can be easily be given a value as small as we like. We must remind at this point that these results are derived starting from the assumption that the auxiliary reactive amplifier metadynes act as if the mutual induction between the stator coils energized by them and the main metadyne circuits were practically negligible. As this assumption is only approximate the results here above derived are also only approximate.

A characteristic feature of this course is the study of each element of the metadyne system separately so as to prepare the background for the « Combinatory ». The element considered in a section may be readily combined with the elements considered in other sections and as a rule it may be embodied into most kinds of metadynes. The development carried out in this section is an example of this rule.

2.) F *generator metadyne*.

This special type of generator metadyne is usually driven at a constant speed, n, by a prime mover whose power is limited at a predetermined value which must not be exceeded. Such a prime mover is any constant torque machine revolving at an arbitrarily defined constant speed. A gas engine and a Diesel engine are such prime movers; a series excited dynamo or a separately excited dynamo with a fixed number of ampere turns; inserted into a constant current loop, are further examples.

Under these conditions when the resistant torque exceeds the above mentioned constant torque value, the speed tends to increase and the inverse occurs when the resistant torque decreases. Thus the F generator metadyne is provided with a speed sensitive device which modifies the control currents when a small discrepancy of the speed, n, appears. A regulator dynamo, already described, in « Metadyne Statics » is such a device endowed further with an easy setting of the desired value of the speed n.

Fig. 24 shows the simplified scheme of this special metadyne having four equidistant brushes, a, b, c, d, any series winding inserted between the brushes and the corresponding terminals A, B, C, D, provided with a secondary variator winding, 1, traversed by the control current I_γ and coupled with a regulator dynamo $R\,D$, whose current, I_ρ traverses a secondary regulator winding, 2. The regulator dynamo is series excited and

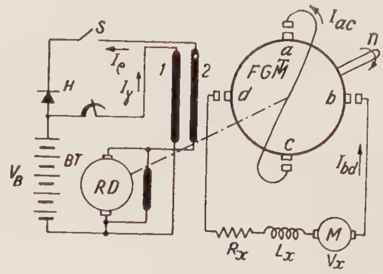


Fig. 24.

connected to a d.c. source, say a battery $B\,T$, through the regulator winding; a device H, is inserted into the regulator current circuit permitting to said current, I_ρ, to flow only in the negative direction, i.e. causing the regulator dynamo to develop an accelerating torque.

The ampere turns created by positive values of I_γ and I_ρ have same direction.

The metadyne, $F\,G\,M$ is, generally, reactive, and for a constant value of I_γ and for $I_\rho = 0$ it will supply a practically constant current until saturation is reached as characteristic $a\,b\,c\,d\,e$ shows in the diagram of fig. 25. In the same diagram, $p\,b\,q\,d\,r$ is an equilateral hyperbola whose power corresponds to the maximum power that the prime mover must supply.

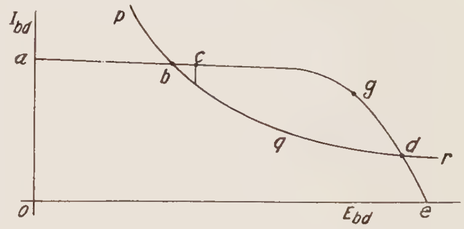


Fig. 25.

As long as the representative point slides along the segment $a\,b$, the current I_ρ, if permitted to flow, would be positive. When the representative point reaches b, the current I_ρ is zero and for increasing values of E_{bd} it becomes negative, it traverses winding 2 and the representative point slides along the hyperbolic segment $b\,q\,d$. At point d again I_ρ becomes zero and for further increasing values of E_{bd}, the representative point slides along segment de.

The fact that at points b and d the direction of I_ρ is inverted passing through zero, may be utilized for acting an apparatus H and cause it to operate as a discriminating valve.

Let us now write the equation of the dynamic operation.

There are four unknowns: I_{ac}, I_{bd}, I_ρ and the speed n (in revolutions per second). Let us call n_0 the critical speed of the regulator dynamo, for which speed there is $I_\rho = 0$; the polar inertia of the rotating member of the F.G.M. and the other rotating members coupled to it; Q_n the nominal torque of same metadyne.

The investigation described in « Metadyne Statics » concerning the operation of the regulator dynamo leads

us to adopt, with a satisfactory approximation, following linear relation for the asymptotic value of the regulator current:

$$I_\rho = C\,(n - n_0). \tag{13}$$

Formulae expressing the torque, given in « Metadyne Statics » have the form of an algebraic homogeneous polynomial of the second order with respect the metadyne currents, hence with respect our unknowns.

Let us assume that for $t < 0$ the switch S is open interrupting the circuit traversed by the regulator current I_ρ and that the representative point operation is at c on the diagram of fig. 25, the speed of the machine being $n_1 < n_0$ and the currents I_{ac}, and I_{bd} having the values $I_{ac}\,(t = 0)$ and $I_{bd}\,(t = 0)$ respectively. For $t = 0$ the switch S is closed and a transient phenomenon starts giving rise to following heavisidean currents: I_ρ; $I_{ac} - I_{ac}\,(t = 0)$; $I_{bd} - I_{bd}\,(t = 0)$; further the speed starts increasing from n_1 towards n_0 accelerating the rotating members of the system.

The dynamic equations expressing the behavior of currents I_{ac}, I_{bd}, I_ρ and I_γ, in their respective circuits are:

$$\begin{aligned}
&\left(R_\pi + L_p \frac{d}{dt}\right) I_{ac}(t) + \left(n\,K_\sigma + L\frac{d}{dt}\right) I_{bd}(t) - \\
&\quad - K_{ac}^{\rho}\, I_\rho(t) = n\,K_{ac}^{\gamma}\, I_\gamma(t) \\
&\left(n\,K_\pi - L\frac{d}{dt}\right) I_{ac}(t) - \left(R_\sigma + L_s\frac{d}{dt}\right) I_{bd}(t) - \\
&\quad - L_{bd}^{\gamma}\frac{d}{dt} I_\gamma(t) - L_{bd}^{\rho}\frac{d}{dt} I_\rho(t) + \\
&\quad + V_x(t) = 0 \\
&V_B - n\,K_\rho\, I_\rho(t) - \left(R_\rho + L_\rho \frac{d}{dt}\right) I_\rho(t) - \\
&\quad - L_\rho^{\gamma}\frac{d}{dt} I_\gamma(t) - L_\rho^{bd}\frac{d}{dt} I_{bd}(t) = 0 \\
&V_B - \left(R_\gamma + L_\gamma\frac{d}{dt}\right) I_\gamma(t) - L_\gamma^{\rho}\frac{d}{dt} I_\rho(t) - \\
&\quad - L_\gamma^{bd}\frac{d}{dt} I_{bd}(t) = 0.
\end{aligned} \tag{14}$$

In order to write the dynamical equation describing the behavior of the speed, n, we must express the electromagnetic torque, Q developed by the $G\,F\,M$:

$$\begin{aligned}
Q = \frac{1}{2\pi}\Big[&- I_{ac}\,(K_{ac}^{\gamma} I_\gamma + K_{ac}^{\rho} I_\rho - S_{ob}^{ac} I_{ac} + \\
&+ S_{od}^{bd} I_{bd}) + (S_{oc}^{bd} I_{bd} - S_{oc}^{ac} I_{ac})\, I_{bd}\Big].
\end{aligned} \tag{15}$$

Where symbols S_{oc}^{ac} and S_{od}^{bd} correspond to compensators and S_{ob}^{ac} and S_{od}^{bd} correspond to stabilizers.

Let us decompose each current, for instance the current I_{ac}, into the value $I_{ac}\,(t = 0)$ it had for $t < 0$, and the heavisidean additional component $I'_{ac}(t)$ starting at $t = 0$:

$$I_{ac}(t) = I_{ac}\,(t = 0) + I'_{ac}(t)$$

and assume that the representative point c is near to the representative point s when the variation of the speed n practically occurs.

Under these conditions the second component $I'_{ac}(t)$ is very small as compared to the first component $I_{ac}\,(t=0)$ and if we neglect the former, the expression (15) in view of (13), simplifies as follows:

$$Q = A\,(n_0 - n) - B \tag{16}$$

where A and B are constants, the constant A has a positive value. Thus the equation describing the movement of the rotating masses may be written as follows:

$$2\pi J \frac{dn}{dt} + F\,n = A\,(n_0 - n) - B + M \tag{17}$$

where the second term of the first member corresponds to friction and great part of iron losses, and where M is the positive, practically constant, torque of the prime mover.

The time constant of the exponential of the solution of equation (17) is:

$$T = 2\pi \frac{J}{-A - F}. \tag{18}$$

In order to estimate the numerical value of this time constant we may revert to texts on normal metadynes. We find thus that:

1.) The friction and iron losses at nominal speed absorb 2 to 6 per cent of the nominal pover; as an average we may take $F\,n_0 = \dfrac{4}{100}\,Q_N$;

2.) A simple regulator dynamo is able to create the nominal torque for a speed discrepancy of 3% to 1% of the nominal speed while an accurately planned one, reduces said speed discrepancy to 0,5% and less; as an average we may thus take:

$$A \frac{2}{100} n_0 = Q_N;$$

3.) When starting a metadyne under the action of its nominal torque, it takes 2″ to 5″ for the medium size machines rating 2 to 50 kW at 1800 R/m and about 15″ for larger ones rating 300 kW at 1000 R/m; as an average we may take 5″.

Neglecting friction and iron losses during the starting period, the equation becomes, for said period:

$$2\pi J \frac{dn}{dt} = Q_N$$

from which we obtain the starting time:

$$t_s = \frac{2\pi J\, n_0}{Q_N} = 5''. \tag{19}$$

Let us multiply both members of the fraction (18) by n_0 and neglect F with respect to $-A$, we obtain in view of the here above three test results:

$$T = \frac{5''}{50} = \frac{1''}{10}.$$

Yet the time constants of a well planned metadyne circuit are of ten to hundred times smaller. We may therefore apply the method of the « practical time duration » developed in the previous chapter and separate equation (17) from equation (14), considering only one variable, the speed n and substituting the asymptotic, constant values to all other variables. Same

reason justifies the assumption here above accepted that when the modification of the speed is practically performed, the currents I_{bd} and I_{ac} have already reached the values corresponding to the representative point on the hyperbola.

On the other hand the movement of the representative point along the hyperbolic segment $b\,s\,q\,d$ is generally very slow as compared with the rapidity of adjustment of the electrical and mechanical variables considered here above and therefore the assumption made that the asymptotic values of the variables are practically reached when the representative point changes its position on an appreciable amount, is correct.

Thus equation (17) is practically corresponding to the real modification of the speed n and its solution where is figuring an exponential with the time constant (18) describes the phenomenon with a good approximation.

Remains the solution of the system of four equations (14) where the speed n may be considered as a constant equal to n_0 without making an appreciable error.

Let us modify the electrical arrangement of the stator windings by using an amplifier metadyne $A\,T$ as fig. 26 shows. Three variator windings are controlling the amplifier metadyne, a variator winding 1 traversed by current I_γ, a second variator windings, 2, traversed by the regulator current I_ρ, and a third variator winding, 3, shunt connected to the secondary terminals of the main metadyne. The amplifier metadyne is energizing

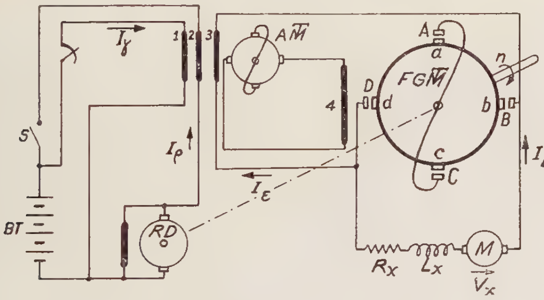


Fig. 26.

the stator winding, 4, of the $F\,G\,M$. The regulator dynamo, $R\,D$, is shunt excited. The addition of the shunt winding, 3, permits modifications of the basic characteristic $a\,b\,c\,g\,d\,e$ adequating it better to the purpose of each particular case.

The use of said shunt winding and the shunt excitation of the regulator dynamo could involve a system of six dynamical equations if the amplifier metadyne were not used. By adopting the scheme of fig. 26 and accepting the assumption that the interposition of the reactive amplifier metadyne practivally eliminates the action of mutual induction between the currents controlling the amplifier metadyne and the brush currents of the main metadyne, and finally arranging for a very small time constant of the circuit traversed by the low intensity currents controlling the amplifier metadyne, we may determine the dynamic behavior of the currents in the main metadyne circuits, with a good approximation, with only two equations.

Let us again consider the transient phenomenon that follows the abrupt closing of switch S at the instant $t = 0$. We have:

$$\left(R_\pi + L_p \frac{d}{dt}\right) I_{ac}(t) + \left(n\,K_\sigma + L \frac{d}{dt}\right) I_{bd}(t) = \tag{20}$$
$$= n\,K_{ac}^\gamma\, I_\gamma(t) + n\,K_{ac}^\rho\, I_\rho(t) + n\,K_{ac}^\varepsilon \frac{1}{R_\varepsilon} \cdot \left[\left(R_x + L_x \frac{d}{dt}\right) I_{bd}(t) - V_x(t)\right]$$

$$\left(n\,K_\pi - L \frac{d}{dt}\right) I_{ac}(t) - \left(R_\sigma + L_s \frac{d}{dt}\right) I_{bd}(t) = \tag{21}$$
$$= -\,V_x(t).$$

In the above system of two differential equations are figuring three unknowns I_{ac}, I_{bd} and I_ρ. This apparent abnormality is due to the application of the method of the « practical duration »; complying with said method we replace the variable $I_\rho(t)$ by the constant $I_\rho(t = 0)$ and the unknowns are again reduced to two. In fact we have assumed that the practical duration of the vanishing transient component of all the control currents of the amplifier metadyne is much smaller than the practical duration of the vanishing transient component of the main metadyne brush currents.

Note that we may apply the principle of superposition of the solutions because equations (20) and (21) are linear; we may then decompose each current into the value it had for $t < 0$, which is known and the additional value of the transient heavisidean current that appears after $t = 0$; let us indicate it by same symbol with an accent. The Laplace transforms of the equations relative to said heavisidean currents are:

$$(R_\pi + \tau\,L_p)\, I'_{ac}(\tau) + (n\,K_\sigma + \tau\,L)\, I'_{bd}(\tau) =$$
$$= n\,K_{ac}^\rho \frac{A\,(n_0 - n)}{\tau} + \tag{22}$$
$$+ n\,K_{ac}^\varepsilon \frac{1}{R_\varepsilon} \left[(R_x + \tau\,L_x) \cdot I'_{bd}(\tau) + V'_x(\tau)\right]$$
$$(n\,K_\pi - \tau\,L)\, I'_{ac}(\tau) - (R_\sigma + \tau\,L_s)\, I'_{bd}(\tau) =$$
$$= -\,V'_x(\tau).$$

In the first of these equation the current I_γ does not appear because it remains constant, at the same value it had for $t < 0$; on the other hand remembering that the practical time duration of the vanishing transient of current I_ρ is much smaller than the one corresponding to the currents I_{ac} and I_{bd}, we readily find that in order to comply with the method based on the «practical duration», the Laplace transform of the term $n\,K_{ac}^\rho\, I_\rho(t)$ is $n\,K_{ac}^\rho \dfrac{A\,(n_0 - n)}{\tau}$.

In fact in order to simulate an important disturbance, we will assume that the speed n is lower than n_0 and the regulator current I_ρ is taking at $t = 0$ its asymptotic value corresponding to equation (13), and remains, thereafter, constant.

The determinant of the coefficients of the unknowns, equated to zero gives:

$$\begin{vmatrix} R_\pi + \tau\,L_p & n\,K_\sigma - n\,K_{ac}^\varepsilon \dfrac{R_x}{R_\varepsilon} + \tau\left(L - n\,K_{ac}^\varepsilon \dfrac{L_x}{R_\varepsilon}\right) \\ n\,K_\pi - \tau\,L & -\,(R_\sigma + \tau\,L_s) \end{vmatrix} = 0. \tag{23}$$

The roots are readily determined and easily commented. Let us assume, for an example, that

$$L - n\,K_{ac}^\varepsilon \frac{L_x}{R_\varepsilon} = 0 \tag{24}$$

then there is:

$$\varrho_1,\ \varrho_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a = L_p L_s$

$$b = L_p R_\sigma + L_s R_\pi - L\left(n K_\sigma - n K^\varepsilon_{ac}\frac{R_x}{R_\varepsilon}\right)$$

$$c = R_\pi R_\sigma + n K_\pi\left(n K_\sigma - n K^\varepsilon_{ac}\frac{R_x}{R_\varepsilon}\right).$$

The action of the shunt winding 3 on the stability of the system is easily recognized.

Equation:

$$R_\pi R_\sigma + n K_\pi\left(n K_\sigma - n K^\varepsilon_{ac}\frac{R_x}{R_\varepsilon}\right) = 0$$

determines the border of the stability domain; if the first member of the last equation is positive we are within the stability domain and out of it if said member is negative.

3.) D *Generator metadyne.*

It has four equidistant brushes per cycle, a, b, c, d, as fig. 27 shows, and through each pair of corresponding terminals, A, C and B, D it energizes two external circuits independently from one another. The external circuit, shown in fig. 27, connected to terminals A, C, is characterized by resistance R_x, self inductance L_x and a source of electromotive force V_x; the circuit connected to terminals B, D, is characterized by R_y, L_y and V_y.

The stator windings comprise any kind of series windings symbolically indicated by the dots opposite each brush, two shunt windings having their magnetic axis coinciding with the primary commutating axis, one connected to the primary terminals and traversed by current I_ε, the other connected to the secondary terminals and traversed by current I_η; further, two shunt windings having their magnetic axis coinciding with the secondary commutating axis, one connected to the primary terminals and traversed by current I_ζ and the other connected to the secondary terminals and traversed by current I_θ; finally two variator windings, one primary, traversed by I_γ and a secondary, traversed by I_δ.

The mechanical shaft is assumed to rotate at constant speed, n, and to be able to develop any required torque, positive or negative.

The corresponding dynamic equations are six and lead obviously to laborious calculations.

Fig. 27 shows two reactive amplifier metadynes $AM\,1$ and $AM\,2$ energizing a Wheatstone bridge stator winding through its four terminals α, β, γ, and δ and creating ampere turns having their magnetic axis coinciding with the primary and the secondary commutating axis correspondingly. The first amplifier metadyne is

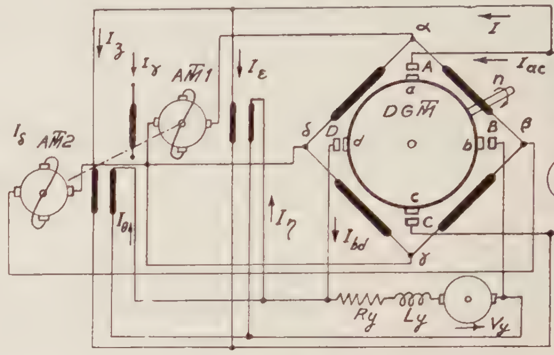


Fig. 27.

controlled by the three currents I_γ I_ε and I_η, the second by the three currents I_δ, I_ζ, I_θ, complying thus with the arrangement here above described. As the power necessary for energizing each of these control windings is very small, we may admit that the resistance inserted in their circuits is so large, with respect to their self and mutual induction, as to justify the assumption that their self and mutual induction can be neglected, or more exactly that their time constant, if their cirduits is considered separately, is very small as compared to that of the metadyne armature circuits.

Under these conditions and taking advantage of the approximate results of section 7 of Chapter I, the dynamic behavior of the brush currents, I_{ac} and I_{bd} is described with a fair approximation by following system of two Laplace transformed equations:

$$(R_\pi + \tau L_p) I_{ac}(\tau) + (n K_\sigma + \tau L) I_{bd}(\tau) = n K^\gamma_{ce} I_\gamma(\tau) + n K^\varepsilon_{ac}\frac{1}{R_\varepsilon}\cdot \tag{25}$$
$$\cdot\left[-V_x(\tau) + (R_x + \tau L_x) I_{ac}(\tau)\right] + n K^\eta_{ac}\frac{1}{R_\eta}\left[-V_y(\tau) + (R_y + \tau L_y) I_{bd}(\tau)\right]$$

$$(n K_\pi - \tau L) I_{ac}(\tau) - (R_\sigma + \tau L_s) I_{bd}(\tau) = n K^\delta_{bd} I\delta(\tau) + n K^\zeta_{bd}\frac{1}{R_\zeta}\left[V_x(\tau) - (R_x + \tau L_x) I_{ac}(\tau)\right] + \tag{26}$$
$$+ n K^\theta_{bd}\frac{1}{R\theta}\left[V_y(\tau) - (R_y + \tau L_y) I_{bd}(\tau)\right].$$

Let us shift at the first member the terms where the two unknown currents, I_{ac} and I_{bd} are figuring, the determinant of their coefficients equated to zero, gives:

$$\begin{vmatrix} R_\pi - n K^\varepsilon_{ac}\dfrac{R_x}{R_\varepsilon} + \tau\left(L_p - n K^\varepsilon_{ac}\dfrac{L_x}{R_\varepsilon}\right) & n K_\sigma - n K^\eta_{ac}\dfrac{R_y}{R_\eta} + \tau\left(L - n K^\eta_{ac}\dfrac{L_y}{R_\eta}\right) \\ n K_\pi + n K^\zeta_{bd}\dfrac{R_x}{R_\zeta} - \tau\left(L - n K^\zeta_{bd}\dfrac{L_x}{R_\zeta}\right) & -\left[R_\sigma - n K^\theta_{bd}\dfrac{R_y}{R\theta} + \tau\left(L_s - n K^\zeta_{bd}\dfrac{L_y}{R\theta}\right)\right] \end{vmatrix} = 0. \tag{27}$$

Let us put:

$$a = R_\pi - n\,K^{\varepsilon}_{ac}\frac{R_x}{R_\varepsilon}\;;\quad b = L_p - n\,K^{\varepsilon}_{ac}\frac{L_x}{R_\varepsilon}$$

$$c = n\,K_\sigma - n\,K^{\eta}_{ac}\frac{R_y}{R_\eta}\;;\; d = L - n\,K^{\eta}_{ac}\frac{L_y}{R_\eta}$$

(28)

$$e = n\,K_\pi + n\,K^{\zeta}_{bd}\frac{R_x}{R_\zeta}\;;\, f = -\left(L - n\,K^{\zeta}_{bd}\frac{L_x}{R_\zeta}\right)$$

$$g = -\left(R_\sigma - n\,K^{\theta}_{bd}\frac{R_x}{R\theta}\;;\; h = -\left(L_s - n\,K^{\theta}_{bd}\frac{L_y}{R\theta}\right)\right.$$

the characteristic equation may be written as follows:

(29) $$\tau^2\,(b\,h - df) + \tau\,(a\,h + b\,g - c\,f + d\,e) + $$

$$+ a\,g - c\,e = 0.$$

We may set arbitrarily the resistances R_ε, R_η, R_ζ, R_θ and hence give values to the three coefficients of equation (29) and thus obtain any desired value for its roots, in other words any desired value of the two time constants.

If $b\,h - df = 0$ in the solution will figure a single exponential with following time constant:

$$\frac{a\,h + b\,g - c\,f + d\,e}{c\,e - a\,g}.$$

If further, the coefficient $a\,h + b\,g - c\,f + d\,e$ is caused to tend to zero through positive values while $c\,e - a\,g$ remains positive and larger than a positive quantity, the time constant tends to zero, and the systems tend to behave dynamically as a network constructed with resistors having negligible inductances.

In particular we may cause the coefficients b, d, f, h to vanish and thus virtually devoid the main metadyne of any self and mutual induction.

The static equations are:

$$a\,I_{ac} + c\,I_{bd} = n\,K^{\gamma}_{ac}\,I_\gamma + n\,K^{\varepsilon}_{ac}\frac{V_x}{R_\varepsilon} + n\,K^{\eta}_{ac}\frac{V_y}{R_\eta}$$

$$e\,I_{ac} + g\,I_{bd} = n\,K^{\delta}_{bd}\,I\delta - n\,K^{\zeta}_{bd}\frac{V_x}{R_\zeta} - n\,K^{\theta}_{bd}\frac{V_y}{R\theta}$$

permitting a great number of combinations.

We will here consider an example; let us add to amplifier $A\,M\,1$ another secondary variator traversed by current I_δ and add to amplifier $A\,M\,2$ another secondary variator traversed by current I_γ and let us indicate the corresponding coefficients by K^{δ}_{ac} and K^{γ}_{bd}. Let us further restrict the case to external passive circuits. The corresponding static equations are:

$$a\,I_{ac} + c\,I_{bd} = n\,(K^{\gamma}_{ac}\,I_\gamma + K^{\delta}_{ac}\,I_\delta)$$

$$e\,I_{ac} + g\,I_{bd} = n\,(K^{\gamma}_{bd}\,I_\gamma + K^{\delta}_{bd}\,I_\delta)$$

and the solution is:

$$I_{ac} = \frac{1}{D}[(K^{\gamma}_{ac}\,g - K^{\gamma}_{bd}\,e)\,I_\gamma + (K^{\delta}_{ac}\,g - K^{\delta}_{bd}\,c)\,I\delta]\,n$$

(29')

$$I_{bd} = \frac{1}{D}[(K^{\gamma}_{bd}\,a - K^{\gamma}_{ac}\,e)\,I_\gamma + (K^{\delta}_{bd}\,a - K^{\delta}_{ac}\,e)\,I\delta]\,n$$

where $D = a\,g - e\,c$.

We may determine the new variator winding in such a way as to satisfy following equations:

$$g\,K^{\delta}_{ac} - c\,K^{\delta}_{bd} = 0$$

(30)

$$a\,K^{\gamma}_{bd} - e\,K^{\gamma}_{ac} = 0$$

without renouncing to the obtained results regarding the dynamic behavior, results which bind the coefficients a, b, c, d, e, f, g, h, because the newly introduced coefficients K^{δ}_{ac} and K^{γ}_{bd} do not figure in the determinant (27) and may be freely utilized for satisfying equations (30). Under these conditions the primary current will depend only on I_γ and the secondary current only on I_δ, in other words we may control the current traversing the two external circuits independently from one another.

4.) *Alpha motor metadyne.*

In this chapter we are considering the dynamic behavior of single metadyne units independently, as much as possible, from the dynamic properties of other machines connected with said unit.

We consider the metadyne brush currents as main unknowns and having separated the terms of the equations which are expressed through the unknowns from the other terms, we have investigated the determinant of the coefficients of the unknowns. By using the symbols R_π and R_σ we took into account the action of possible stabilizing or stimulating windings belonging to external circuits with respect to the metadyne unit, but if we want to consider the stability of the units, in itself, we must restrict the nature of the external circuits so as to eliminate as much as possible any stabilizing or stimulating action due to external apparatus. Such an appropriated apparatus is, for instance, a low resistance, full compensated, dynamo, or a low resistance battery, All cases of metadyne generators examined in the previous chapters may be readily restricted as here above indicated.

Considering now metadyne motors we are lead to assume a well determined source supplying the power to the motor. The most simple sources are characterized by a constant voltage or a constant current. As a constant voltage source, a fully compensated, low resistance dynamo separately excited, capable of an output very large with respect the nominal power of the motor, a large battery, and even a large alternator would be appropriated.

But a source of an unidirectional or of an alternating current of constant intensity, implies the existance of stabilizing actions keeping constant this intensity and this assumption excludes or at least hampers the possibility of investigating the stability of the considered unit in itself.

We may overcome the difficulty by remembering that a motor may generally operate as a generator and inversely a generator may also absorb power; thus we may investigate the dynamic behavior of a motor metadyne inserted in a series distribution system, by causing it to supply power to the external circuit and by eliminating as completely as we can any stabilizing or stimulating action of the external circuit.

An Alpha motor metadyne is a unit particularly-contrived for being inserted into a loop of a series distribution and we will assume that the external part of the loop consists of a source having no stabilizing nor stimulating action, inducing an e.m.f. V_x and having a negligible resistance and self inductance, as fig. 28 schematically shows.

The Alpha motor metadyne is described in Metadyne Statics as endowed with a family of very useful torque-speed characteristics; it comprises essentially two stator

windings, a secondary hypercompensator, 2, and a secondary variator winding, 1. Into the secondary circuit a rheostat, 3, is inserted controlling the no load speed, n_0, of the motor, a critical point separating the speed range into two segments, one segment comprising the

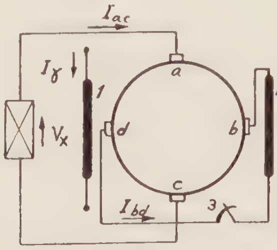


Fig. 28.

zero speed, corresponding to a torque developed in one direction and another segment with the torque in the opposite direction. The variator winding 1 controls the direction and the magnitude of the starting torque and the rheostat 3 controls the value of the no load speed, n_0. Fig. 29 shows a typical torque-speed characteristic of an Alpha motor.

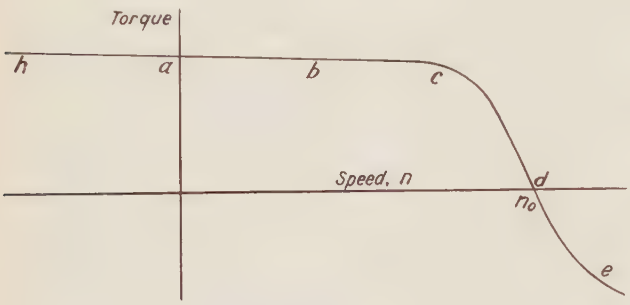


Fig. 29.

Let us assume that voltage V_x and current I_γ, traversing winding 1, are heavisidean quantities reaching for $t = 0$ the value I_γ and remaining constant after wards. The Laplace transformed equations are:

$$(31)\quad \begin{aligned} &(R_p + \tau L_p)\, I_{ac}(\tau) + \\ &\quad + [n(K - K^\beta_{ac}) + \tau L]\, I_{bd}(\tau) = \\ &= \left[n\, K^\gamma_{ac}\, I_\gamma + V_x \right] \frac{1}{\tau} \\ &(n\,K - \tau\,L)\, I_{ac}(\tau) - (R_s + \tau\,L_s)\, I_{bd}(\tau) = 0. \end{aligned}$$

The determinant of the coefficients of the unknowns equated to zero gives following characteristic equations:

$$a\,\tau^2 + b\,\tau + c = 0$$

where

$$(32)\quad \begin{aligned} a &= L_p\, L_s - L^2 \\ b &= L_p\, R_s + L_s\, R_p + n\,L\,K^\beta_{ac} \\ c &= R_p\, R_s + n^2\, K\,(K - K^\beta_{ac}) \end{aligned}$$

The coefficient K^β_{ac} refers to the hypercompensator 2 and it is essentially positive and larger than K; in other words we have: $K_\sigma = K - K^\beta_{ac} < 0$.

In case there is: $L_p\,R_s + L_s\,R_p + n\,L\,K^\beta_{ac} > 0$.

The critical equation is:

$$(33)\qquad R_p\, R_s + n^2\, K\,(K - K^\beta_{ac}) = 0$$

and it defines the critical values of the parameters R_s, n, K^β_{ac} separating the domain of stability from the domain of instability.

R_s is variable and it is of the same order as R_p when the no load speed, n_0, is required to be very low. Then the product $R_p\,R_s$ is very small as compared with $n^2\,K^2$; thus the domain of instability is entered even for a very small modulus of the reactivity resistance R_r which is imaginary:

$$R^2_{bd} = n^2\,K\,(K - K^\beta_{bd}).$$

When the no load speed, n_0, increases, R_s increases proportional to n_0 but the negative term of the critical equation is proportional to the square of the speed and we may conclude that the Alpha motor metadyne with the schema corresponding to fig. 28, is, generally, an unstable unit with a rather strong stimulating action.

We are naturally led to substitute a primary stabilizing, winding 4, to the secondary variator winding, as the schema of fig. 30 shows. The control of the value of the starting torque is then obtained by chosing the number of turns of the primary stabilizer as schematically shown by 10; an inverter 9 inverts the direction of the torque.

Under these conditions the global resistance R_π, sum of ohmic resistance R_p of the primary circuit and the kinetic resistance R_k corresponding to stabilizer,

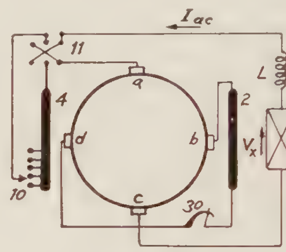


Fig. 30.

4, substitutes R_p in the critical equation (33), reducing energically the global stimulating action of the motor and, possibly rendering the operation stable at any speed; in fact the product $R_\pi\,R_s$ is almost proportional to n^2 as the square of the reactivity resistance.

Fig. 31 shows the schema of another variant: a primary amplifier 5 with a variable number of turns is

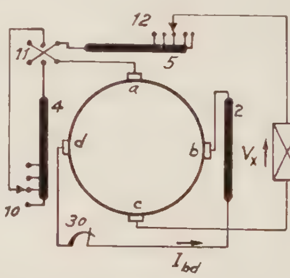


Fig. 31.

added permitting the control of a wider range of possible values for the no load speed, n_0. The addition of the primary amplifier, 5, increases the modulus of the imaginary reactivity resistance R_r.

The variants of fig. 30 and 31 have the inconvenience of requiring the somehow cumbersome switchgear 10,

11 and 12 which handle the primary current. The primary amplifier 5 may be replaced by a primary variator winding, but then an external source is again needed.

5.) *Delta motor metadyne.*

It comprises essentially four equidistant brushes per cycles, a primary pair of diametrically opposite brushes and a secondary pair; the primary pair is connected to the external source through a primary compensator, 6, as the schema of fig. 32 shows; the secondary pair is connected to a resistor 3 through a secondary hyper compensator, 2; it comprises a secondary variator winding, 1, traversed by current I_γ, and a possible primary variator winding, 7, traversed by current I_δ.

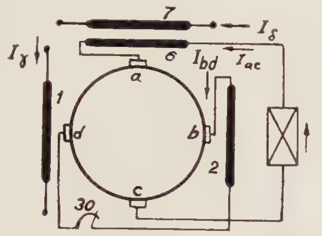


Fig. 32.

Assuming that the voltage V_x of the source and the current I_γ and I_δ of the variator windings are heavisidean for $t = 0$, and remain constant for $t > 0$, the Laplace transformed equations are:

$$(34)\quad \begin{aligned} &(R_p + \tau\, L_p)\, I_{ac}(\tau) + [n\,(K - K^\beta_{ac}) + \tau\, L]\, I_{bd}(\tau) = \\ &= [n\, K^\gamma_{ac}\, I_\gamma + V_x]\, \frac{1}{\tau} \\ &[n\,(K - K^\alpha_{bd}) - \tau\, L]\, I_{ac}(\tau) - [R_s + \tau\, L_s]\, I_{bd}(\tau) = \\ &= n\, K^\delta_{bd}\, I_\delta\, \frac{1}{\tau}\,. \end{aligned}$$

Coefficients K^β_{ac} and K^α_{bd} refer to windings 2 and 6 respectively, and the characteristic equation of the determinant of the coefficients of the unknows equated to zero is:

$$a\,\tau^2 + b\,\tau + c = 0$$

where

$$(35)\quad \begin{aligned} a &= L_p\, L_s - L^2 \\ b &= L_p\, R_s + L_s\, R_p + n\, L\,(K^\beta_{ac} - K^\alpha_{bd}) \\ c &= R_p\, R_s + n^2\,(K - K^\alpha_{bd})\,(K - K^\beta_{ac})\,. \end{aligned}$$

In case following condition depending on the speed, n, is satisfied:

$$(36)\quad L_p\, R_s - L_s\, R_p + n\, L\,(K^\beta_{ac} - K^\alpha_{bd}) > 0$$

then the critical equation separating the domain of stability from the domain of instability, is as follows:

$$(37)\quad R_p\, R_s + n^2\,(K - K^\alpha_{bd})\,(K - K^\beta_{ac}) = 0.$$

In equation (37) we may consider as parameters following quantities: R_s which becomes larger the larger must be the no load speed, n_0; the speed n; the two coefficients K^α_{bd} and K^β_{ac}, where $K^\beta_{ac} > K$.

If the primary compensator is compensating 100% of the armature reaction, then the first member of equation (37) reduces to:

$$R_p\, R_s > 0$$

and we are in the domain of stability, whatever may be the degree of compensation of the secondary compensator 2.

In this case the critical equation becomes the one that separates the positive values of b from the negative ones, i.e. following:

$$(38)\quad L_p\, R_s + L_s\, R_p + n\, L\,(K^\beta_{ac} - K) = 0.$$

In fact then there is $K = K^\alpha_{bd}$.

As the resistance R_s is large, both equations (37) and (38) leave a fair margin of stable operation for values of the speed lower than the ones that satisfy the two critical equations, (37) and (38). The possible addition of stabilizers increases said margin within the stability domain, and we may conclude that a skillful calculation will allow a reduction of the compensating degree of the primary stabilizer large enough for creating a primary magnetic flux sufficient to induce between the secondary brushes the e.m.f. needed for developing, at the desired no load speed n_0, the ampere-turns necessary to practically compensate the ampere-turns of the secondary winding, 1. In this case winding 7 is no more necessary but it remains always useful for better controlling the value of the no load speed n_0.

Fig. 33 shows the schema of a variant arrangement of the Delta motor metadyne. The main machine is shown provided with a primary compensator, 6, whose degree of compensation is lower, generally, or equal, in the extreme case, to 100% and with a Wheatstone bridge type of stator winding 14, 14, energized through its primary terminals α and γ, by amplifier metadyne A 1 and through its secondary terminals, β and δ, through amplifier metadyne A 2. The latter creates ampereturns for a flux having their magnetic axis winding with the secondary commutating axis and the former amplifier

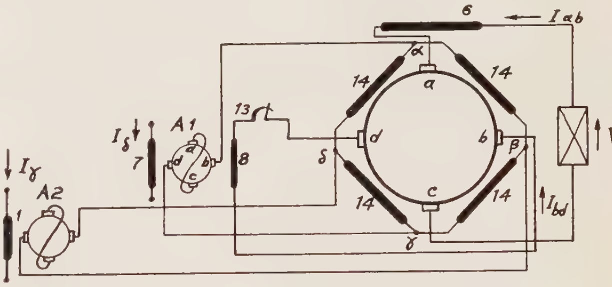


Fig. 33.

metadyne creates ampere-turns for a primary flux. Amplifier metadyne A 1 is shown controlled by current I_δ, traversing winding 7 and by current I_{bd}, traversing winding 8 and set by rheostat 13; amplifier metadyne A 2 is shown controlled by current I_γ traversing winding 1.

Under the usual assumption that control windings 1, 7, 8 require such a little power that we may neglect their self and mutual induction because of the large resistance included in their circuits, and further that the amplifier metadynes are very reactive, we may write the Laplace transformed dynamic equations as in (34) and comment them in a similar way with the advantage that the resistance R_s is now much larger than for the case of fig. 31. We may thus reduce the degree of compensation of the primary compensator. If this degree may be lowered to 0, winding 6 may be dispensed with and the main metadyne is then reduced to an Alpha motor; if the degree of compensation of winding 6 is

low enough, the control through current I_δ is not indispensable and then the amplifier A 2 may be eliminated.

If the primary compensator 6 is deemed necessary or useful we may incorporate it into the Wheatstone bridge type winding 14 by adding a third control variator winding to amplifier A 1, traversed by the primary current or by a current proportional to it.

The static torque-speed characteristics of the Alpha and the Delta motor metadynes inserted in a loop of a series distribution system, are very similar.

We found that the Alpha motor is generally unstable and that the Delta motor or the Alpha motor provided with auxiliary amplifier reactive metadyne are, frequently, stable.

But motors in a series distribution loop are energized usually by metadynes which can impress stability even to machines in itself unstable, and then we may allow a limited stimulation action to the motors without impairing the overall operation and the here above found unstable, in itself, motors, may be adopted for a satisfactory operation.

6.) *Theta and Gamma motors.*

The order of these motors is also $m = 4$ the brushes being equidistant; the primary pair of brushes is connected to a source and the two secondary brushes are essentially short-circuited. The essential stator windings are a primary variator winding and a secondary one; the latter may be replaced by a primary stabilizer.

The Gamma motor differs from the Theta motor by the presence of a primary compensator not existing in the Theta motor.

Comparing the Alpha and Delta motors to the Gamma and Theta motors, we may say that the former are characterized by a secondary hypercompensator, the resistance of the secondary circuit of the first pair of motors being generally large with respect to the armature resistance while in the Theta and Gamma motors the hypercompensator does not exist and the resistance of the secondary circuit is often practically equal to the armature resistance.

On the other hand the Delta and the Gamma motors have a primary compensator which does not exist in the Alpha and the Theta motors; the latter two motors may have, on the contrary a primary amplifier.

We will consider in this section simultaneously both Theta and Gamma motors taking into account the here above stated difference of their stator winding arrangement.

Fig. 34 and 35 show the simplified schema of a Theta and a Gamma motor respectively; the secondary brushes b, d, are shortcircuited, there is a primary variator

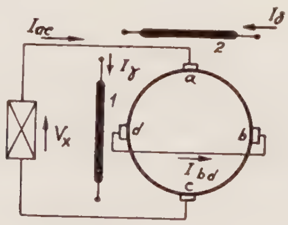


Fig. 34.

winding, 2, traversed by the control current I_δ and a secondary variator winding, 1, traversed by the control current I_γ. The Gamma motor is further provided with a primary compensator 3, missing on the Theta motor.

The primary brushes a, c, of the motors may be connected to a shunt or a series distribution system; but for the particular purpose of investigating the dynamic behavior of the motors, we will assume that the source of power energizing the distribution systems is simply characterized by the creation of a voltage V_x in the positive direction of the primary circuit.

Let us assume that the control currents I_γ and I_δ and the voltage V_x are heavisidean beginning at $t = 0$ and remain constant thereafter, then the Laplace transformed equations are:

$$(R_p + \tau L_p)\, I_{ac}(\tau) + n\, K\, I_{bd}(\tau) =$$

$$(39) \qquad = (V_x + n\, K^{\gamma}_{ac}\, I_\gamma)\, \frac{1}{\tau}$$

$$n\,(K - K^{\alpha}_{bd})\, I_{ac}(\tau) - (R_s + \tau L_s)\, I_{bd}(\tau) = 0$$

where the coefficient K^{α}_{bd} refers to compensator 3 and is essentially positive.

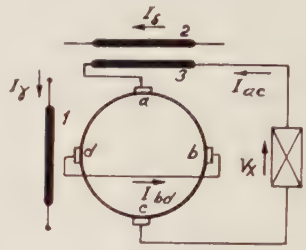


Fig. 35.

The equation of the determinant of the coefficients of the unknowns I_{ac}, and I_{bd}, equated to zero is:

$$a\,\tau^2 + b\,\tau + c = 0$$

where

$$a = L_p\, L_s$$

$$(40) \qquad b = L_p\, R_s + L_s\, R_p$$

$$c = R_s\, R_p + n^2\,(K - K^{\alpha}_{bd})\, K.$$

Thus as long as the first member of the critical equation:

$$R_s\, R_p + n^2\,(K - K^{\alpha}_{bd})\, K = 0$$

is positive, we are into the stability domain. Yet the compensator is generally an hypocompensator and we may conclude that Theta and Gamma motors have a stabilizing action and particularly the Theta motor has a strong one.

In order to avoid the controlling currents, I_γ, we may replace winding 1 by a primary stabilizing winding 4 completed with an inverter 10 and a switchgear 11 permitting to modify the number of its turns, as fig. 36

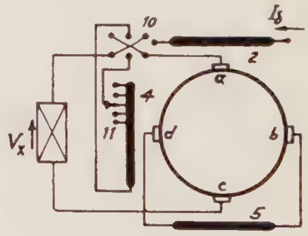


Fig. 36.

and 37 show. Similarly, in case of a Gamma motor we may replace the variator winding, 2, by a primary compensator, 3, with a variable number of turns to be set by switchgear 12.

To the inconvenience of having a rather heavy switch-

gear must be added the stimulating action that winding 4 will develop when the motor is braking.

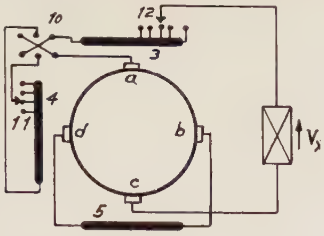


Fig. 37.

Fig. 38 shows the complete schema of a variant of a Gamma motor metadyne.

Two stator windings are shown, a secondary stabilizing winding, 5 and a Wheatstone type winding 9,9,9,9, energized at its two primary terminals, α and γ, by an amplifier metadyne A 1, and at its two secondary terminals, β and δ, by an amplifier metadyne A 2. On the shaft of the Gamma motor a shunt excited regulator dynamo $R\,D$ is coupled, the critical speed of which is set by rheostat 10; it supplies current I_ρ.

Amplifier A 1 is controlled by three secondary variators, 2, 3 and 7 traversed by currents I_δ, I_ρ and I_{ac} respectively; amplifier A 2 is controlled by variators 1 and 8 traversed by currents I_γ and I_ρ.

Assuming, as usually, that the currents traversing the secondary variator windings of the amplifiers have an

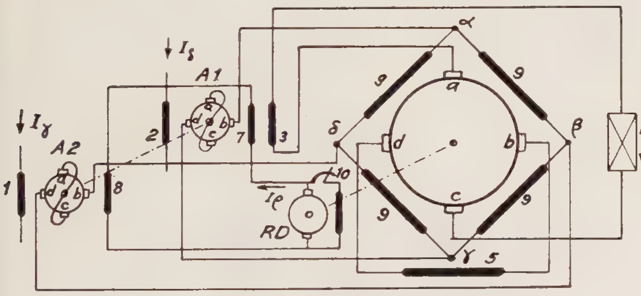


Fig. 38.

intensity independent from the derivatives with respect time of the currents traversing the other variator windings and the armature of the amplifiers, and further assuming that the amplifiers are very reactive, we obtain following Laplace transformed equations describing the dynamic behavior of the system, all control currents and voltage V_x being heavysidean:

$$(41)\quad \begin{aligned} &(R_p + \tau L_p)\, I_{ac}(\tau) + (n\,K + \tau\,L)\, I_{bd}(\tau) = \\ &\qquad = [V_x + n\,K^{\gamma}_{ac}\, I_\gamma + n\,K^{\rho}_{ac}\, I_\rho]\,\frac{1}{\tau} \\ &(n\,K_\pi - \tau\,L)\, I_{ac}(\tau) - (R_\sigma + \tau\,L_s)\, I_{bd}(\tau) = \\ &\qquad = -[n\,K^{\delta}_{bd}\, I_\delta + n\,K^{\rho}_{bd}\, I_\rho]\,\frac{1}{\tau}\,. \end{aligned}$$

The corresponding determinant of coefficients of the unknows equated to zero gives following characteristic equation:

$$a\,\tau^2 + b\,\tau + c = 0$$

where

$$(42)\quad \begin{aligned} a &= L_p\, L_s - L^2 \\ b &= L_p\, R_\sigma + L_s\, R_p + L\,n\,(K_\pi - K) \\ c &= R_p\, R_\sigma + n^2\, K\, K_\pi\,. \end{aligned}$$

As $K_\pi \leq K$ for the Gamma motor and $K_\pi = K$ for the Theta motor, and as L is negative here, as the reader will readily recognize, the critical equation becomes following one:

$$(43)\quad R_p\, R_g + n^2\, K\, K_\pi = 0$$

and hence we are always within the domain of stability.

The static equations are followings:

$$(44)\quad \begin{aligned} &R_p\, I_{ac} + n\,K\, I_{bd} = V_x + n\,K^{\gamma}_{ac}\, I_\gamma + n\,K^{\rho}_{ac}\, I_\rho \\ &n\,K_\pi\, I_{ac} - R_\sigma\, I_{bd} = -n\,K^{\delta}_{bd}\, I_\delta - n\,K^{\rho}_{bd}\, I_\rho \\ &I_\rho = k\,(n - n_0) \text{ with the limitation } I_\rho > 0. \end{aligned}$$

Thus for all values of the speed, n, lower than n_0, we have: $I_\rho = 0$.

Thus when $n < n_0$ and the sources is a constant voltage source we obtain following solution:

$$(45)\quad \begin{aligned} I_{ac} &= \frac{R_\sigma\,(V_x + n\,K^{\gamma}_{ac}\, I_\gamma) - n^2\, K\, K^{\delta}_{bd}\, I_\delta}{R_p\, R_\sigma + n^2\, K\, K_\pi} \\ I_{bd} &= \frac{R_p\, n\, K^{\delta}_{bd}\, I_\delta + n\,K_\pi\,(V_x + n\,K^{\gamma}_{ac}\, I_\gamma)}{R_p\, R_\sigma + n^2\, K\, K_\pi}\,. \end{aligned}$$

The characteristic representing I_{ac} is shown in fig. 39. The segment $k\ a\ b\ c$ is cubic having an asymptote parallel to the axis of the speed, n, whose ordinate is:

$$(46)\quad \frac{K^{\beta}_{bd}\, K^{\gamma}_{ac}\, I_\gamma - K\, K^{\delta}_{bd}\, I_\delta}{K\,(K - K^{\alpha}_{bd})}$$

and having a cusp at k, on the axis of current I_{ac}, whose ordinate is

$$(47)\quad V_x\,\frac{1}{R_p}\,.$$

In formula (46) the coefficients K^{β}_{bd} and K^{α}_{bd} correspond to windings 5 and 3 respectively.

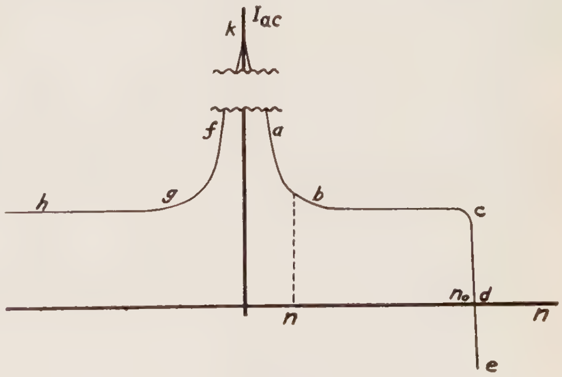


Fig. 39.

When $n > n_0$ and the source is a constant voltage source, we obtain following solution:

$$(48)\quad \begin{aligned} I_{ac} &= \frac{R_\sigma\,(V_x + n\,K^{\gamma}_{ac}\, I_\gamma) - n^2\, K\,[K^{\delta}_{bd}\, I_\delta + K^{\rho}_{bd}\, k(n - n_0)]}{R_p\, R_\sigma + n^2\, K\, K_\pi} \\ I_{bd} &= \frac{R_p\, n\, K^{\delta}_{bd}\, I_\delta + n\,K_\pi\,[V_x + n\,K^{\gamma}_{ac}\, I_\gamma + n\,K^{\rho}_{ac}\, k\,(n - n_0)]}{R_p\, R_\sigma + n^2\, K\, K_\pi} \end{aligned}$$

The characteristic representing I_{ac} is again a cubic which around the no load speed, n_0, is very near to a straight line almost vertical, as the segment $c\,d\,e$ of fig. 39 shows.

By setting I_γ and I_δ we may shift the cubic represented by the first equation (45) so as to have its almost horizontal segment $b\,c$, at any desired distance from the axis of the speed. In other words along this segment, $b\,c$, the Gamma motor will operate at almost constant intensity of the primary current with any desired positive value, accelerating and absorbing power from the line, or with any desired negative value, braking and recuperating electric power to the line.

For a speed $n < n_s$ corresponding to a point near point b, the intensity of the primary current must be reduced to the desired value by means of a starting rheostat.

When the distribution, in which the Gamma motor is inserted, is a series one energized by a source supplying a current of constant intensity, the variables of sistem (48) are I_{bd}, V_x and I_p and for $n < n_0$ the solution is:

(49)
$$I_{bd} = \frac{1}{R_\sigma}\left[n K_\pi I_{ac} + n K_{bd}^{\delta} I\delta\right]$$
$$V_x = I_{ac} R_p - n K_{ac}^{\gamma} I_\gamma + \frac{n K}{R_\sigma}\left[n K_\pi I_{ac} + n K_{bd}^{\delta} I\delta\right]$$

and the torque is given by:

(50)
$$T = \frac{1}{2\pi n} I_{ac} V_x = \frac{-1}{2\pi} K_{ac}^{\gamma} I_\gamma I_{ac} + n \frac{K}{2\pi K_{bd}^{\beta}}\left[K_\pi I_{ac}^2 + K_{bd}^{\delta} I\delta\, I_{ac}\right].$$

This formula gives the torque on the shaft of the motor when we neglect the Joule effect losses in the primary and in the secondary circuit.

By chosing a constant value for I_γ and by setting I_δ proportional to I_{ac} we may reduce formula (50) to following form

(51)
$$T = A + n B$$

where A and B may take any positive or negative value. In other words for $n < n_0$ the Gamma motor may start, accelerate or brake with any desired torque being a linear function of the speed n and, in particular, being constant. The diagram of fig. 40 shows an acce-

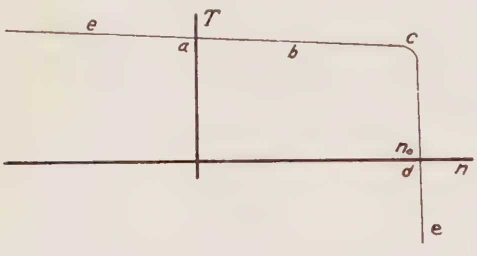


Fig. 40.

lerating torque slightly decreasing with the speed n, along all the segment $a\,b\,c$, or a braking torque slightly increasing with the speed along the segment $a\,h$.

For a constant current series distribution and for $n > n_0$, the torque, when the losses due to Joule effect are neglected, is given by following formula:

(52)
$$T = \frac{-I_{ac}}{2\pi}\left[K_{ac}^{\gamma} I_\gamma + K_{ac}^{\rho} k\,(n - n_0) + n \frac{K}{K_{bd}^{\beta}}\left(K_\pi I_{ac} + K_{bd}^{\delta} I\delta + K_{bd}^{\rho} k\,(n - n_0)\right)\right]$$

and around the critical speed it creates a sharp bent as shown in fig. 40 with a segmental characteristic $c\,d\,e$ very near to a straight line almost horizontal.

The characteristics of fig. 39 and 40, adapted for accelerating and for braking and obtained with the motor operating well within the stability domain, are remarkable for their practical utility.

7.) *Metadyne transformers; the «Cross Transformer».*

Metadyne transformers are, generally, more elaborate than metadyne generators and motors but the investigation of their dynamic behavior is readily accomplished by referring to the investigation already carried out for the generators.

We will chose three examples: a «Cross» transformer, a «Caduceus» transformer and a «Saturn» transformer.

Fig. 41 shows the schema of a «Cross» transformer metadyne having four equidistant brushes per cycle, a, b, c, d, the primary pair of brushes a, c, being connected to an external circuit the primary external circuit represented by a resistor, R_x, a self induction, L_x, and a source of a kinetic electromotive force, V_x, and the secondary pair of brushes, b, d, being connected to the secondary external circuit characterized by a R_y, L_y and V_y.

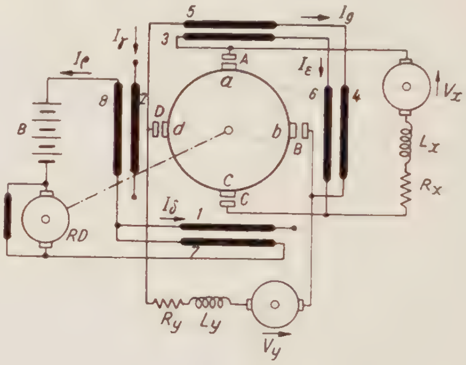


Fig. 41.

On the shaft of the transformer a regulator dynamo, $R\,D$, is coupled, supplying current I_p. The transformer is shown comprising any series stator winding symbolically represented by the two blocks at the brushes location, and eight other stator windings: primary and secondary variator windings 1 and 2 traversed by I_δ and I_γ respectively; primary and secondary regulator windings, 7 and 8 traversed by the regulator current I_p; a primary and a secondary shunt winding, 3 and 6 connected across the primary brushes and a primary an a secondary shunt windings, 5 and 4, connected across the secondary brushes.

We will assume that stator windings 1, 2, 3, 4, 5, 6, 7 and 8 are fused in a single, Wheatstone bridge type winding energized by two amplifier metadynes connected and controlled in the well known manner, so as to reproduce same stator ampere turns as the above mentioned windings; we will assume further that the resistance of the controlling circuits of the amplifier metadynes is very large and that said amplifiers are very reactive. Under these conditions the Laplace

transform of the dynamic equations may be reduced to the following ones:

$$(R_\pi + \tau L_p) I_{ac}(\tau) + (n K_\sigma + \tau L) I_{bd}(\tau) =$$

$$= [n K_{ac}^{\gamma} I_\gamma + n K_{ac}^{\rho} I_\rho + V_x] \frac{1}{\tau} +$$

$$+ n K_{ac}^{\varepsilon} I_\varepsilon(\tau) + n K_{ac}^{\zeta} I_\zeta(\tau)$$

(53)

$$(n K_\pi - \tau L) I_{ac}(\tau) - (R_\sigma + \tau L_s) I_{bd}(\tau) =$$

$$= -[n K_{bd}^{\delta} I_\delta + n K_{bd}^{\rho} I_\rho + V_y] \frac{1}{\tau} -$$

$$- n K_{bd}^{\varepsilon} I_\varepsilon(\tau) - n K_{bd}^{\zeta} I_\zeta(\tau)$$

where we have assumed, as usually, that currents I_γ and I_δ and voltages V_x and V_y are heavisidean; where currents I_ε and I_δ are traversing the shunt windings connected across the primary and the secondary brushes, respectively, satisfying following Laplace transformed equations:

$$R_\varepsilon I_\varepsilon(\tau) + (R_x + \tau L_x) I_{ac}(\tau) = V_x \frac{1}{\tau}$$

(54)

$$R_\zeta I_\zeta(\tau) + (R_y + \tau L_y) I_{bd}(\tau) = V_y \frac{1}{\tau}$$

Thus the unknowns are reduced to two, $I_{ac}(\tau)$ and $I_{bd}(\tau)$ and the determinant of their coefficients equated to zero yields following characteristic equation:

(55) $$\begin{vmatrix} A_1 + \tau B_1 & A_2 + \tau B_2 \\ A_3 + \tau B_3 & A_4 + \tau B_4 \end{vmatrix} = 0$$

where:

(56)
$$A_1 = R_\pi + n K_{ac}^{\varepsilon} \frac{R_x}{R_\varepsilon};$$

$$A_2 = n K_\sigma + n K_{ac}^{\zeta} \frac{R_y}{R_\zeta};$$

$$A_3 = n K_\pi - n K_{bd}^{\varepsilon} \frac{R_x}{R_\varepsilon};$$

$$A_4 = - R_\sigma - n K_{bd}^{\zeta} \frac{R_y}{R_\zeta};$$

The characteristic equation may take following form:

$$a \tau^2 + b \tau + c = 0$$

whose roots are:

$$\varrho_1, \varrho_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where

(57)
$$a = B_1 B_4 - B_2 B_3$$
$$b = A_1 B_4 + A_4 B_1 - (A_2 B_3 + A_3 B_2)$$
$$c = A_1 A_4 - A_2 A_3$$

The critical equation is:

(58) $$A_1 B_4 + A_4 B_1 - (A_2 B_3 + A_3 B_2) = 0$$

and in case the first member of (54) is positive, the critical equation becomes

(59) $$(A_1 A_4 - A_2 A_3) = 0.$$

We may consider the constants:

(60) $$K_{ac}^{\varepsilon} \frac{1}{R_\varepsilon} \quad K_{ac}^{\zeta} \frac{1}{R_\zeta}, \qquad K_{bd}^{\varepsilon} \frac{1}{R_\varepsilon} \quad K_{bd}^{\zeta} \frac{1}{R_\zeta}$$

as four parameters to which we may give any desired value, positive or negative.

In order to remain well within the stability domain we need to render positive the first member of equations (58) and (59), and this constitutes two vincula to be respected by our four parameters, much less limiting that the satisfaction of two equations, and leaves wide latitude for disposing of said parameters for modulating the static characteristics.

We may also accept a limitation of the freedom to chose the four parameters (60) for satisfying equation (61):

(61) $$B_1 B_4 - B_2 B_3 = 0$$

and then the only root of the characteristic equation is:

(62) $$\varrho = \frac{A_2 A_3 - A_1 A_4}{A_1 B_4 + A_4 B_1 - (A_2 B_3 + A_3 B_2)}.$$

Rendering this root negative constitutes only a furher vinculum on our parameters and much freedom is still left for shaping the static characteristics. In this case the whole system behaves, dynamically, as a single circuit provided with resistance and self induction or with resistance and elastance.

We may chose to satisfy following equation:

(63) $$A_1 B_4 + A_4 B_1 - (A_2 B_3 + A_3 B_2) = 0$$

$$B_1 = L_p + n K_{ac}^{\varepsilon} \frac{L_x}{R_\varepsilon};$$

$$B_2 = L + n K_{ac}^{\zeta} \frac{L_y}{R_\zeta};$$

$$B_3 = - L - n K_{bd}^{\varepsilon} \frac{L_x}{R_\varepsilon};$$

$$B_4 = - L_s - n K_{bd}^{\zeta} \frac{L_y}{R_\zeta};$$

in which case the roots of the characteristic equations are:

(64) $$\varrho_1, \varrho_2 = \pm \sqrt{\frac{A_2 A_3 - A_1 A_4}{B_1 B_4 - B_2 B_3}}$$

and if the quantity under the radical is caused to be negative we will have a permanent, sinusoidal current, operation with a pulsation equal to the modulus of quantity (64).

If following relations are satisfied:

(65)
$$A_1 A_4 - A_2 A_3 = 0$$
$$\frac{A_2 B_3 + A_3 B_2}{B_1 B_4 - B_2 B_3} > 0$$

we shall have again a stable operation and the metadyne will dynamically behave like a simple circuit having resistance and a single energy reserve.

Finally we may determine the four parameters (60) by following system of equations:

$$B_1 = 0; \; B_2 = 0; \; B_3 = 0; \; B_4 = 0; \tag{66}$$

and then the metadyne will dynamically behave as a network constitute by mere resistors without self-inductance nor capacity.

Having considered the many ways for obtaining a stable operation, let us now take up briefly the nature of the static characteristics.

The static equations are:

$$(67) \quad \begin{aligned} & R_\pi I_{ac} + n K_\sigma I_{bd} = n K^\gamma_{ac} I_\gamma + n K^\rho_{ac} I_\rho + V_x + n K^\varepsilon_{ac} \frac{R_x I_{ac} + V_x}{R_\varepsilon} + n K^\zeta_{ac} \frac{R_y I_{bd} + V_y}{R_\zeta} \\ & n K_\pi I_{ac} - R_\sigma I_{bd} = - n K^\delta_{bd} I_\delta - n K^\rho_{bd} I_\rho - V_y - n K^\varepsilon_{bd} \frac{R_x I_{ac} + V_x}{R_\varepsilon} - n K^\zeta_{bd} \frac{R_y I_{bd} + V_x}{R_\zeta} \\ & P_L + I_{ac} V_x + I_{bd} V_y + R_p I_a^2 + R_s I_b^2 = 0 \end{aligned}$$

where P_L is the sum of the very small losses due to the control currents and to the iron and mechanical losses. The system (67) being of the second degree, with respect brush currents and external voltages, we shall obtain, for constant values of parameters I_γ and I_δ, conic characteristics between brush currents and external voltages, which may degenerate into linear characteristics between an external voltage and the corresponding brush current and a conic characteristic between the other external voltage and its corresponding current; this, finally, may further degenerate into linear characteristics between said variables.

It is important to note that we dispose of two more parameters, $n K^\rho_{ac}$ and $n K^\rho_{bd}$ for shaping the static characteristics; these two parameters do not figure in the characteristic equation and, hence, neither in the critical equations.

The discussion of the most usual of these cases is developed in metadyne statics.

The dynamic equations in case currents I_γ, I_δ, I_ρ and voltages V_x and V_y are sinusoidal of the same pulsation, have same terms functions of the two brush currents and lead to same characteristic and same critical equations.

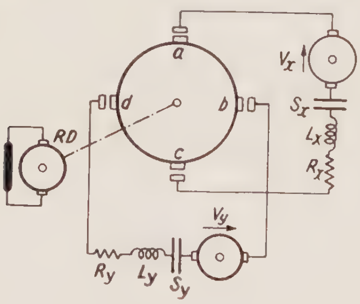


Fig. 42.

But if the external circuit comprise capacitors as the partial schema of fig. 42 shows, then the characteristic equation becomes of the form:

$$a \tau^3 + b \tau^2 + c \tau + d = 0$$

too laborious to be commented in its general form; numerical solutions for each special case may then be carried out.

In « Metadyne Periodic » the static characteristic of many similar cases is considered and we may say that, frequently, the operation is stable whenever the static characteristic shows well defined currents for from singular points for which the amplitude of the currents tend towards infinite.

8.) *The Caduceus transformer.*

A variant scheme is represented by fig. 43.

The metadyne order is still $m = 4$, i.e. there are four brushes per cycle and they are equidistant from one another but the armature bears two armature windings, primary and secondary, isolated from one another and associated with separate commutators. The primary brushes, a, c, bear on the primary commutator associated with the primary winding; the secondary brushes b, d, collect the current traversing the secondary winding.

The dots A, B, C, D, opposite the brushes symbolize the possible existance of any kind of series winding.

Terminals C and D are connected together and to terminal T_1 of one of the external circuits character-

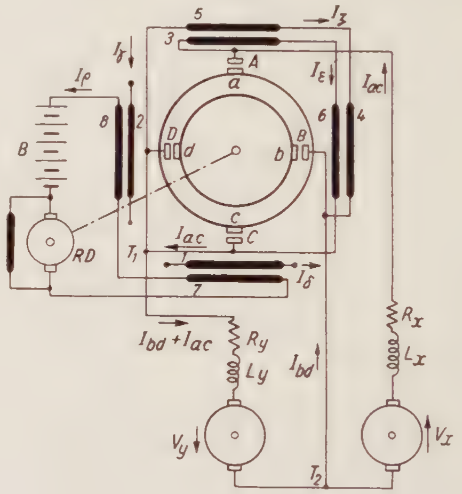


Fig. 43.

ized by resistance R_y, inductance L_y and electromotive force V_y; the other terminal, T_2 of same external circuit is connected to brush B, and it forms also one terminal of other external circuit characterized by R_x, L_x, and V_x; finally terminal A is the other terminal of the last indicated external circuit.

On the armature shaft is coupled a regulator dynamo $R\,D$.

Fig. 43 shows eight non series stator windings indicated by same numeral as in fig. 41, and having same connections and purpose.

Accepting same assumptions as for the Cross transformer metadyne of fig. 41, we may write the Laplace

transform of the dynamic equations as follows:

$$
\begin{aligned}
&(R_\pi' + \tau L_p) I_{ac}(\tau) + (n K_\sigma + \tau L) I_{bd}(\tau) = -[R_y + R_x + \tau(L_x + L_y)] I_{ac}(\tau) + (V_y + V_x)\frac{1}{\tau} + \\
&\qquad -[R_y + \tau L_y] I_{bd}(\tau) + n K_{ac}^{\gamma} I_\gamma \frac{1}{\tau} + n K_{ac}^{\rho} I_\rho \frac{1}{\tau} + n K_{ac}^{\varepsilon} I_\varepsilon(\tau) + n K_{ac}^{\zeta} I_\zeta(\tau) \\
&(n K_\pi - \tau L) I_{ac}(\tau) - (R'_\sigma + \tau L_s) I_{bd}(\tau) = (R_y + \tau L_y)[I_{ac}(\tau) + I_{bd}(\tau)] - V_y \frac{1}{\tau} + \\
&\qquad - n K_{bd}^{\delta} I_\delta \frac{1}{\tau} - n K_{bd}^{\rho} I_\rho \frac{1}{\tau} - n K_{bd}^{\varepsilon} I_\varepsilon(\tau) - n K_{bd}^{\zeta} I_\zeta(\tau) \\
&R_\varepsilon I_\varepsilon(\tau) = (V_x + V_y)\frac{1}{\tau} - [R_x + R_y + \tau(L_x + L_y)] I_{ac}(\tau) - (R_y + \tau L_y) I_{bd}(\tau) \cdot \\
&R_\zeta I_\zeta(\tau) = V_y \frac{1}{\tau} - (R_y + \tau L_y)[I_{ac}(\tau) + I_{bd}(\tau)].
\end{aligned}
\tag{68}
$$

Where R_π' and R_σ' are here the ohmic resistances and the kinetic resistances *inside* the metadyne terminals.

The characteristic equation is:

$$a\tau^2 + b\tau + c = 0$$

where

$$
\begin{aligned}
a &= B_1 B_4 - B_2 B_3 \\
b &= A_1 B_4 + A_4 B_1 - (A_2 B_3 + A_3 B_2) \\
c &= A_1 A_4 - A_2 A_3
\end{aligned}
$$

the capital letters of the last equations having following value:

$$
\begin{aligned}
A_1 &= R_\pi' + R_x + R_y - n K_{ac}^{\varepsilon} \frac{R_x + R_y}{R_\varepsilon} - \\
&\quad - n K_{ac}^{\zeta} \frac{R_y}{R_\zeta} \\
B_1 &= L_p + L_x + L_y - n K_{ac}^{\varepsilon} \frac{L_x + L_y}{R_\varepsilon} - \\
&\quad - n K_{ac}^{\zeta} \frac{L_y}{R_\zeta} \\
A_2 &= n K_\sigma + R_y - n K_{ac}^{\varepsilon} \frac{R_y}{R_\varepsilon} - n K_{ac}^{\zeta} \frac{R_y}{R_\zeta} \\
B_2 &= L + L_y - n K_{ac}^{\varepsilon} \frac{L_y}{R_\varepsilon} - n K_{ac}^{\zeta} \frac{L_y}{R_\zeta} \\
A_3 &= n K_\pi + R_y - n K_{bd}^{\varepsilon} \frac{R_x + R_y}{R_\varepsilon} - \\
&\quad - n K_{bd}^{\zeta} \frac{R_y}{R_\zeta} \\
B_3 &= - L - L_y + n K_{bd}^{\varepsilon} \frac{L_x + L_y}{R_\varepsilon} + \\
&\quad + n K_{bd}^{\zeta} \frac{L_y}{R_\zeta} \\
A_4 &= - R_\sigma' - R_y + n K_{bd}^{\varepsilon} \frac{R_y}{R_\varepsilon} + n K_{bd}^{\zeta} \frac{R_y}{R_\zeta} \\
B_4 &= - L_s - L_y + n K_{bd}^{\varepsilon} \frac{L_y}{R_\varepsilon} + n K_{bd}^{\zeta} \frac{L_y}{R_\zeta}.
\end{aligned}
\tag{69}
$$

The comments may now be developed parallel to the case concerning the Cross transformer with similar results; the latter are briefly following:

— With the parameters we dispose of, it is always possible to have a dynamical operation well within the stability domain and yet leave a wide margin for shaping the static characteristics;

— We may obtain a stable operation of the metadyne with a dynamic behavior similar to a simple electric circuit, provided with only one potential energy reserve, with fair margin for shaping the static characteristic;

— We may have a stable operation with a sinusoidal current of arbitrarily determined pulsation;

— We may have a stable operation with a dynamic behavior similar to a simple electric circuit characterized by resistances and absence of any potential energy reserve, yet allowing some margin for shaping the static characteristics.

Let us note that whenever one of the voltages, V_{AC} or V_{BD}, between the primary or the secondary terminals, is zero, the transformed power is also zero if the Joule effect inside the metadyne is neglected. Yet the metadyne may confer stability to the whole system even if the external circuits have a stimulating action. We may thus imagine that the metadyne is creating on one side a real power and on the other side a virtual power controlling the stability of the whole system and that those two powers are independent from one another.

9.) *The Saturn transformer.*

Let us finally consider a variant of the Saturn transformer metadyne as indicated by the scheme of fig. 44. The order of this metadyne is $m = 6$ having six brushes per cycle; there are two armature windings,

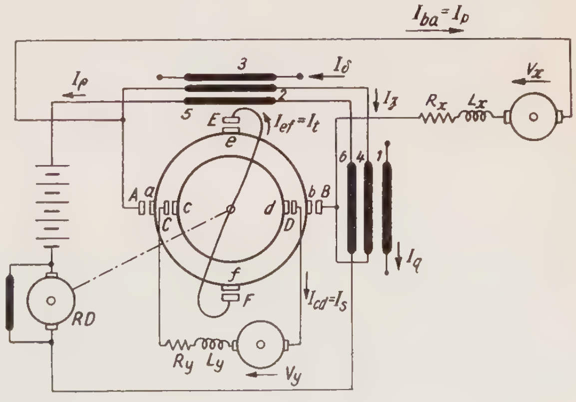


Fig. 44.

isolated from one another and two commutators. An armature winding and its commutator are associated by two diametrical opposite brushes a, c, connected to an external circuit characterized by a resistance R_x, inductance L_x and an e.m.f. V_x; we will refer at this armature winding, commutator, pair of brushes and external circuit, as «primary». The other armature winding and its commutator are associated with two diametrical opposite brushes b, d, connected to another external circuit characterized by R_y, L_y, V_y; all these elements are referred at as «secondary»; the primary and secondary brushes have same commutating axis. Finally the third pair of diametrically opposite brushes e, f refereed at as «tertiary», have their commutating axis perpendicular to the common commutating axis of the primary and the secondary, may bear on any of the two commutators the primary or the secondary and they are essentially short-circuited. The ratio of the total conductors of the two armatures is arbitrarily chosen. Any series stator windings may be applied and their possible presence is symbolically shown in the figure by the terminal blocs A, B, C, D, E, F, opposite the homonymous brushes.

Six other stator windings are shown in the figure, a primary variator winding, 1, traversed by I_ε, a tertiary variator winding, 3, traversed by current I_δ, two shunt windings, 2 and 4 traversed by current I_ζ, and two regulator windings, 5 and 6, traversed by the regulator current I_ρ.

Assuming again that the last mentioned stator windings are fused in only one, energized by two reactive amplifier metadynes whose control windings have a very large resistance as compared with their inductance, we may write the dynamic equations in a simplified form by using following symbols:

I_p, I_s, I_t for currents I_{ba}, I_{cd} and I_{ef} respectively;
L_p, L_s, L_t, L_{ps}, L_{st}, L_{tp} for the self and mutual inductances of the three circuits, primary, secondary and tertiary; $n\,K^p_{gh}$, $n\,K^s_{gh}$, $n\,K^t_{gh}$ for the e.m.f. induced between brushes g, h by unit primary, secondary or tertiary current, respectively, when they traverse the armature;
$n\,K^\alpha_{gh}$, $n\,K^\beta_{gh}$, $n\,K^\gamma_{gh}$ for the e.m.f. induced between brushes g, h by unit primary, secondary or tertiary current respectively, when they traverse stator windings:

$$K^\pi_{gh} = K^p_{gh} + K^\alpha_{gh}; \qquad K^\sigma_{gh} = K^s_{gh} + K^\beta_{gh};$$

$$K^\tau_{gh} = K^t_{gh} + K^\gamma_{gh}.$$

Let us admit currents I_δ, I_ε, I_ρ and voltages V_x and V_y heavisidean at $t = 0$ and constant thereafter; applying second Ohm's low to the primary, secondary and tertiary circuits, we have:

$$(70)\quad \begin{aligned} &(-R_p + n\,K^\pi_{ba} - \tau\,L_p)\,I_p(\tau) + (n\,K^\sigma_{ba} - \tau\,L_{ps})\,I_s(\tau) + (n\,K^\tau_{ba} - \tau\,L_{pt})\,I_t(\tau) + \\ &\qquad + [V_x + n\,K^\delta_{ba}\,I_\delta + n\,K^\rho_{ba}\,I_\rho]\frac{1}{\tau} + n\,K^\zeta_{ba}\,I_\zeta(\tau) = 0 \\ &(n\,K^\pi_{cd} - \tau\,L_{ps})\,I_p(\tau) + (n\,K^\sigma_{cd} - R_s - \tau\,L_s)\,I_s(\tau) + (n\,K^\tau_{cd} - \tau\,L_{st})\,I_t(\tau) + \\ &\qquad + [V_y + n\,K^\delta_{cd}\,I_\delta + n\,K^\rho_{cd}\,I_\rho]\frac{1}{\tau} + n\,K^\zeta_{cd}\,I_\zeta(\tau) = 0 \\ &(n\,K^\pi_{ef} - \tau\,L_{tp})\,I_p(\tau) + (n\,K^\sigma_{ef} - \tau\,L_{st})\,I_s(\tau) + (n\,K^\tau_{ef} - R_t - \tau\,L_t)\,I_t(\tau) + \\ &\qquad + [n\,K^\varepsilon_{ef}\,I_\varepsilon + n\,K^\rho_{ef}\,I_\rho]\frac{1}{\tau} + n\,K^\zeta_{ef}\,I_\zeta(\tau) = 0. \end{aligned}$$

Under the assumptions made, current $I_\zeta(\tau)$ is determined by following equation:

$$(71)\qquad -R_\zeta\,I_\zeta(\tau) + V_y\frac{1}{\tau} - (R_y + \tau\,L_y)\,I_s(\tau) = 0$$

Substituting the value of $I_\zeta(\tau)$ drawn from this equation in the three previous ones we obtain three equations of following form:

$$(72)\quad \begin{aligned} &(A_1' + \tau\,B_1')\,I_p(\tau) + (A_2' + \tau\,B_2')\,I_s(\tau) + (A_3' + \tau\,B_3')\,I_t(\tau) + \\ &\qquad + \left[V_x + V_y\,n\,K^\zeta_{ba}\frac{1}{R_\zeta} + n\,K^\delta_{ba}\,I_\delta + n\,K^\rho_{ba}\,I_\rho\right]\frac{1}{\tau} = 0 \end{aligned}$$

$$(73)\quad \begin{aligned} &(A_1'' + \tau\,B_1'')\,I_p(\tau) + (A_2'' + \tau\,B_2'')\,I_s(\tau) + (A_3'' + \tau\,B_3'')\,I_t(\tau) + \\ &\qquad + \left[V_y\left(1 + n\,K^\zeta_{cd}\frac{1}{R_\zeta}\right) + n\,K^\delta_{cd}\,I_\delta + n\,K^\rho_{cd}\,I_\rho\right]\frac{1}{\tau} = 0 \end{aligned}$$

$$(74)\quad \begin{aligned} &(A_1''' + \tau\,B_1''')\,I_p(\tau) + (A_2''' + \tau\,B_2''')\,I_s(\tau) + (A_3''' + \tau\,B_3''')\,I_t(\tau) + \\ &\qquad + \left[n\,K^\varepsilon_{ef}\,I_\varepsilon + n\,K^\rho_{ef}\,I_\rho + V_y\,n\,K^\zeta_{ef}\frac{1}{R_\zeta}\right]\frac{1}{\tau} = 0. \end{aligned}$$

In these three equations we may consider as unknowns the three currents I_p, I_s, I_t and this will lead us to a characteristic equation of the third degree adequate for investigating the stability of the Saturn metadyne proper.

If we consider instead, the Saturn transformer (fig. 45) as inserted into a loop of a series distribution where the intensity of the current is kept constant by some

source able to impress said law, then the previous equations must be changed into following ones:

(75)
$$A_1' I_p \frac{1}{\tau} + (A_2' + \tau B_2') I_s(\tau) + (A_3' + \tau B_3') I_t(\tau) + V_x(\tau) + \\ + \left[V_y n K_{ba}^{\zeta} \frac{1}{R_\zeta} + n K_{ba}^{\delta} I\delta + n K_{ba}^{\rho} I_\rho \right] \frac{1}{\tau} = 0$$

(76)
$$A_1'' I_p \frac{1}{\tau} + (A_2'' + \tau B_2'') I_s(\tau) + (A_3'' + \tau B_3'') I_t(\tau) + \\ + \left[V_y \left(1 + n K_{cd}^{\zeta} \frac{1}{R_\zeta} \right) + n K_{cd}^{\delta} I\delta + n K_{cd}^{\rho} I_\rho \right] \frac{1}{\tau} = 0$$

(77)
$$A_1''' I_p \frac{1}{\tau} + (A_2''' + \tau B_2''') I_s(\tau) + (A_3''' + \tau B_3''') I_t(\tau) + \\ + \left[n K_{ef}^{\varepsilon} I\varepsilon + n K_{ef}^{\rho} I_\rho + V_y n K_{ef}^{\zeta} \frac{1}{R_\zeta} \right] \frac{1}{\tau} = 0$$

with three following unknowns: $I_s(\tau)$, $I_t(\tau)$ and $V_x(\tau)$. We note that in this case the system of the last three equations is split into a pair of equations (76) and (77) having only two unknowns: $I_s(\tau)$ and $I_t(\tau)$ and into equation (75) which determines $V_x(\tau)$ as a function of $I_s(\tau)$ and $I_t(\tau)$.

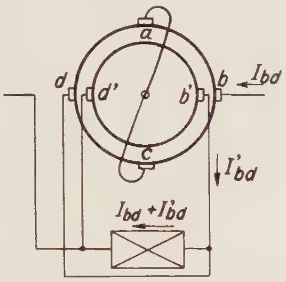


Fig. 45.

Thus in this case again the characteristic equation is reduced to a second degree algebraic equation where we dispose of two arbitrary parameters: $K_{cd}^{\zeta} \frac{1}{R_\zeta}$ and $K_{ef}^{\zeta} \frac{1}{R_\zeta}$. We may here develop comments similar to the ones already known and arrive at similar results.

It is important to note that the commutation of the primary and the secondary currents occur into same slots at the same time and that under special, but in practice most usual conditions, the commutation may become very easy.

10.) *Some general remarks.*

We have frequently admitted that some variator currents, say I_γ, I_δ are heavisidean and remain constant thereafter. In many practical applications they are instead variable functions of time and sometime they are generated by modulators giving to said functions rather elaborated forms. The question arises then how this will interfere with the dynamic equations considered here above.

In many sections and particularly in the last ones, we have applied the « practical duration » rule splitting the system of a great number of dynamic equations into three groups: one group comprising variables whose vanishing transient component has a very short practical duration of the order of $\frac{1''}{10\,000}$, a second group corresponding to an order of $\frac{1''}{100}$ and a third corresponding to an order of $1''$. To the first group belong the controlling currents of the auxiliary amplifier metadynes; to the second group belong the brush current of the main metadyne and to the third group belongs the speed n of same main metadyne.

We were thus able to reduce the fundamental investigation to a set of a few equations, generally two, with the brush currents of the main metadyne as unknowns.

The modulated currents I_γ, I_δ here above considered are usually varying with the speed of mechanical masses of great inertia and causing modifications of currents I_γ and I_δ of say 1% in one second. These currents, on the other hand, traverse circuits characterized by a very short practical duration. We may therefore consider the variations of said currents as belonging to a fourth group corresponding to a practical duration of a higher order than the practical duration corresponding to the speed, n, of the main metadyne. Thus the results already obtained in the sections of this chapter, are not practically modified by the modulation of stator currents such as the one considered here above for I_γ and I_δ.

The examples chosen in the sections of this chapter cover obviously a few cases only of metadynes, but they were chosen such as to allow us to show different ways of approaching and simplifying otherwise complex problems.

Chapter IV.

METADYNE AMPLIFIERS

1.) Cronological note.

During the first world war the author, in military service in the Italian Navy, was working in the « wireles » laboratory of the Navy still under the spell of Marconi's initiative. By that time it was common to create electromagnetic oscillations by means of successive interruptions and reestablishments of a primary circuit which at each reestablishment would yield rapidly damped oscillations. This primary circuit was a

bulky one with hollow conductors of 20 cm diameter and a weel, having a diameter of 2 meters, switching on and of by means of sparks.

The author while improving details of the existing plant, thought that it would be more convenient to adopt the following arrangement:

1.) to have as primary oscillating circuit a low power and small size circuit and then to amplify by using rotational motion as factor of amplification, through an apparatus with a low power excitation circuit and a rapidly rotating, high power armature;

2.) to create in the low power primary oscillating circuit *permanent* and easily modulated oscillations by means of an oscillating system comprising two or more rotating machines inductively interlinked with one another so as to create an oscillating current at a predetermined pulsation.

The suggestion was not carried out, but it has been recorded in an article issued later on when the war was over and the author in civilian duty.

The article has been published in « L'Elettrotecnica » the organ of the Italian association of electrical engineers, on August 1919 pages 481 to 487. From it fig. 46,

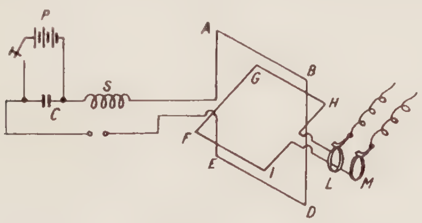


Fig. 46.

47 and 48 are here under reproduced; fig. 46 illustrates the power amplification principle; fig. 47 gives a rough idea of the rotating power amplifier and fig. 48 shows one of the many variant arrangements suggested for creating the primary, permanently oscillating, circuit.

Time has demonstrated that both suggestions found

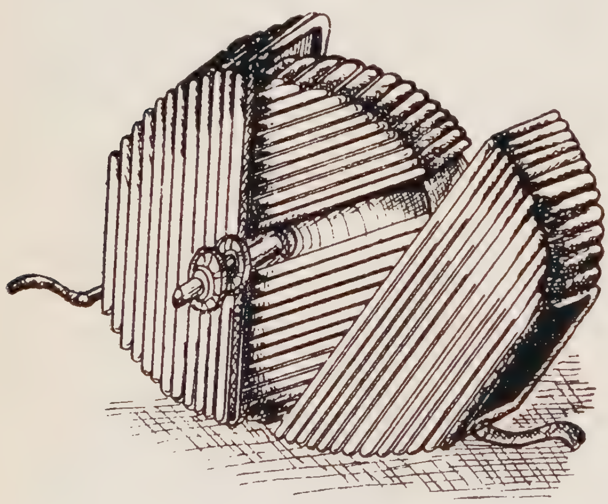


Fig. 47.

wide application. Here we will consider only the principle of the power amplification by means of rapid rotational movement. Fig. 46 shows an amplifier alternator, or an element of an amplifier dynamo.

In 1932 the author was charged by the Italian Navy to calculate a plant for gun control using the metadyne system which had yielded satisfactory results in deck auxiliaries; he then returned to his old principle of power amplification through rapid rotation this time applying it on metadynes because the latter needed an exciting power far smaller than dynamos. Almost all types of metadynes may be used, but the author chose as most convenient and simplest three types which yielded very gratifying results at the tests in 1934.

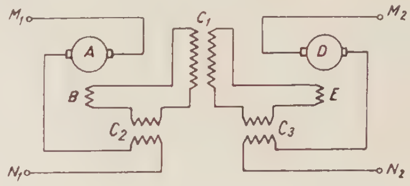


Fig. 48.

The author by that time was in the United States, in Erie, Pennsylvania, consulting to the General Electric Co. and applied metadyne amplifiers to the control of Diesel electric locomotives, an application thousand times since then reiterated.

In 1938 the Italian Navy had already equipped two battle ships with metadyne gun control and the author obtained permission from the Navy to communicate the satisfactory results to his allied companies, and he found authorative men to understand, appreciate and give wide application to metadyne amplifiers, enthusiastic Mr. Fletcher, penetrating Mr. Alger and scientifically skilled Mr. Alexanderson; and later on brilliant Sir Robert Wattson-Watt. Since then amplifier metadynes rendered great services to the defense and to civilian services. Metadyne amplifiers took various commercial names; the author's name was rarely mentioned.

2.) A comparative comment on « amplifiers ».

Amplifiers, are extensively used today and are developing every day more. Some are based on electronic emixion in an almost a vacuum or a relatively thin atmosphere; other on electronic emixion through special solid structures of molecules preferally along a priviledged direction; some are based on the sharp change of the value of permability at a definite magnetic field of ferric magnetic alloys; other on the variable electromagnetic or electrical inductivity of vibrating bodies and finally other on unidirectionally rotating machines.

Al comprise a controlling means involving a relatively small power, and output means involving a relatively large power. Their operation is usually represented in our calculation by a « transfer function » where figure two factors, one expressing the power ratio of the controlling means and of output means, another determining the deformation of the actual output characteristics as compared to the desired ones. Hence the name of « amplifiers ».

The largest number of electronic magnetic amplifiers operate as a throttle to the flow of a current drawing power from an independent source. This justifies the originally given name of « values » and disqualifies the name of « amplifiers » which on the contrary depicts more adequately the operation of the unidirectionally rotating apparatus.

Any unidirectionally rotating machines transforming mechanical power to electrical power and whose output is controlled by an « excitation » may be classified as a possible amplifier if conveniently arranged for this purpose, such as an alternator and, much better, a dynamo; metadynes appear more adequate; in fact we found in « Metadyne Statics » and in the previous chapter that the control of the operation of a metadyne requires a very small power.

Let us define the purpose of an amplifier. For the great majority of cases the purpose may be defined a

S

follows: « a circuit, to which we will refer as pilot circuit, is traversed by a current, i, involving generally a small power, p, and it is derived to obtain an output involving a large power, P, characterized by an output current, I or an output voltage V, proportional at any moment to the pilot current, i ».

We may refer to the ratio P/p as to « power amplification factor », and to a function expressing the discrepancy of the characteristic values the actual output as compared to the desired ones, as to a « fidelity factor » often used in shop parlance. We may reasonably expect that these factors reduce or increase simultaneously.

The advent of metadynes leads us to a generalization of the purpose, because a metadyne is characterized by an output characterized not simply by a voltage or an intensity of current, but by a function correlating the output voltage and the output current

$$f(V, I) = 0$$

where the parameters, practically used for controlling this function are usually more than one.

We may thus formulate following solvable request:

« given a plurality of pilot circuits, traversed by currents i_α, i_β, i_δ, involving generally small powers p_α, p_β, $\ldots p_\delta$, it is desired to obtain an output, characterized by a predetermined function of $f(V, I) = 0$ correlating the output voltage and current involving a large power P ».

We may refer to ratios

$$\frac{dP}{dp_\alpha}, \frac{dP}{dp_\beta}, \ldots \frac{dP}{dp_\delta}$$

and to their sum as to individual and global power amplification factors and to a function expressing the discrepancy of the characteristic values of the actual output as compared to the desired ones as to deformation factors. This is the reason of the name « Conformer metadyne » given by the author at the first amplifier metadynes used for the Italian Navy.

We have here above considered a metadyne output comprising one voltage and one current but we found metadynes having an output comprising a plurality of voltages and currents, and hence the purpose of amplification may be accordingly widened.

Finally we have found metadynes having both in put and output characterized by functions correlating the corresponding voltages and currents; hence a further generalization. These more general cases are considered in a special section of the Metadyne Combinatory concerning computers; here only very simple cases will be dealt with as examples.

3.) The S amplifier metadyne $S\,A\,M\,T$.

It has the general arrangement of an S generator metadyne but it is provided generally with a plurality of independent secondary windings, say three, traversed by pilot currents I_γ, I_δ, I_ε, as fig. 49 shows schematically. In this figure outside the three pilot windings, 1, 2, 3, two other shunt windings are indicated, 4 and 5; the secondary external circuit is represented by a rectangle with two diagonals, 10 and the terminals A B C D opposite the brushes, a, b, c, d remind, symbolically, that there may be any series winding. The choice and the setting of the stator windings depends on the particular application considered.

As a first example let us consider the simplest operation whereby the secondary currents must be a linear homogeneous combination of the pilot currents:

$$I_{bd} = k_\gamma I_\gamma + k_\delta I_\delta + k_\varepsilon I_\varepsilon \tag{1}$$

the coefficients k being independent, as much as possible, the static value of the load and of its variations with

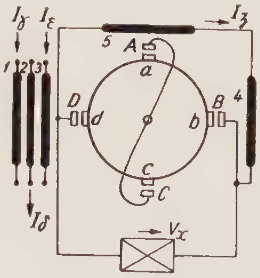


Fig. 49.

respect time and the power amplification factors having a value as high as possible.

The operation may be represented by many proceedings:

— by the unidirectional asymptotic values in function of the load;

— by the variation of some values, assumed sinusoidal, with respect the frequency;

— by the determination of the transient values following a predetermined perturbation;

— by the determination of the practical duration of the vanishing transient components following a premined perturbation;

— or by still other proceedings the here above indicated being the most usual; we may refer to them as the *static response* of the amplifier, the *frequency response*, the *transfer function response* and the *practical duration response* of the amplifier, the three latter being generally referred at as « quick response ».

Let us consider the static equations corresponding to the schema of fig. 49, expressed by means of the same symbols used in the previous chapter.

$$\begin{aligned} R_\pi I_{ac} + n K_\sigma I_{bd} &= \Sigma\, n K_{ac}^\delta I_\delta + \\ &+ n K_{ac}^\zeta \frac{V_x - R_x I_{bd}}{R_\zeta} \\ n K_\pi I_{ac} - R_\sigma I_{bd} &= - V_x - n K_{bd}^\zeta \frac{V_x - R_x I_{bd}}{R_\zeta} \end{aligned} \tag{2}$$

where the symbol of summation reminds that all pilot currents must be considered, where V_x and R_x are the voltage induced by consumer 10 and its resistance and where I_ζ and R_ζ are the current and the resistance of the shunt windings 4 and 5.

The solution is:

$$\begin{aligned} I_{ac} &= \frac{\left(R_\sigma + n K_{bd}^\zeta \frac{R_x}{R_\zeta}\right)\left(\Sigma\, n K_{ac}^\delta I_\delta + n K_{ac}^\zeta \frac{V_x}{R_\zeta}\right) - \left(n K_\sigma + n K_{ac}^\zeta \frac{R_x}{R_\zeta}\right) V_x \left(1 + n K_{bd}^\zeta \frac{l}{R_\zeta}\right)}{R_\pi \left(R_\sigma + n K_{bd}^\zeta \frac{R_x}{R_\zeta}\right) + n^2 K_\pi \left(K_\sigma + K_{ac}^\zeta \frac{R_x}{R_\zeta}\right)} \\ I_{bd} &= \frac{R_\pi V_x \left(1 + n K_{bd}^\zeta \frac{1}{R_\zeta}\right) + n K_\pi \left(\Sigma\, n K_{ac}^\delta I_\delta + n K_{ac}^\zeta \frac{V_x}{R_\zeta}\right)}{R_\pi \left(R_\sigma + n K_{bd}^\zeta \frac{R_x}{R_\zeta}\right) + n^2 K_\pi \left(K_o + K_{ac}^\zeta \frac{R_x}{R_\zeta}\right)} \end{aligned} \tag{3}$$

The coefficient of the perturbing term, where V_x figures as factor in the expression of I_{bd}, is:

$$R_\pi \left(1 + n K_{bd}^{\zeta} \frac{1}{R_\zeta} \right) + n^2 K_\pi K_{ac}^{\zeta} \frac{1}{R_\zeta} . \tag{4}$$

We may reduce this quantity to zero by conveniently determining any of the arbitrary parameters we dispose of in this expression, say, by means of parameter K_{ac}^{ζ}, and thus render the expression of I_{bd} independent of V_x.

Fig. 50 (1) shows an experimental curve obtained at the laboratory of the University of Minnesota; the intensity of I_{bd} remains constant with an approximation of $\pm 0.25\%$ while the external voltage E_{BD} varies from zero to the value corresponding to the nominal power of the machine. In this cases as external voltage was taken the sum $V_x + R_x I_{bd}$.

The coefficients of proportionality between I_{bd} and any pilot current, depends on the iron saturation of the magnetic circuit, local saturation of the corners of the main polar segments, may be eliminated or at least substantially reduced, by adopting a compensation, a primary and a secondary compensation, of the local armature ampere turns under the main polar segments, uniformly distributed along the airgap under these segments.

For the value of the power amplification factor under unidirectional static conditions, we may argue as follows: It is easy to determine the value of the primary current necessary for creating the primary flux inducing the nominal secondary voltage, and hence to determine the necessary ampere-turns, say A_0, that the pilot winding must impress for obtaining said primary current, corresponding to a pilot power p_0 The value of the ratio $\frac{p_0}{P_N}$, where P_N is the nominal power of the machine, oscillates, for the many units calculated by the author, between 10 000 and 100 000 according to the size and nominal speed of the machine.

Yet the pilot winding must develop not only the ampere turns A_0, but also the ampere turns A_r corresponding to the non compensated part of the secondary armature reaction, the ampere turns A_s corresponding to the primary stabilizing winding, and the ampere turns A_q due to increase of the primary current because of a possible partial compensation of the primary armature ampere turns, and because of the presence of the secondary stabilizing winding creating ampere turns antagonistic to the primary current armature ampere-turns; thus the total ampere turns, A, that the pilot winding must develop is the sum of all previous ones:

$$A = A_0 + A_r + A_s + A_q . \tag{5}$$

Quantities A_0, A_s are proportional to the intensity of the primary current; quantity A_r is proportional to the intensity of the secondary current, and quantity A_q is a linear homogeneous, function of both.

The power absorbed by the pilot winding is proportional to the square of the total ampere turns A, hence the interest to reduce terms A_r, A_s, A_q each of which is frequently much higher than A_0.

Term A_r, generally the largest, can be reduced by increasing the degree of compensation of the secondary armature ampere turns and it is reduced to zero when the degree of compensation reaches 100%. But the reduction of the reactivity of the metadyne impairs its quick response and shifts the representative point of its operation in the Gauss plane near the frontier of the stability domain.

Similarly A_s is reduced by reducing the primary kinetic resistance and disappears when there is no primary stabilizer, but the latter is a main factor, for the reduction of the overall practical duration of the vanishing transients of the brush currents.

Term A_q is reduced by eliminating the primary hypocompensator, by adding a primary amplifier, at the expenses of the quick response and the constancy of coefficients k figuring in equation (1).

A compromise is generally accepted, and the power amplification factor for the asymptotic d.c. values corresponding to the nominal power oscillates for practical applications between following values 500 and 5000.

It is important to note that the value of the power amplification varies rapidly with the load and the nature of the function $f(V, I) = 0$ correlating the output voltage and current. Take the losses corresponding to ampere turns A_0, they vary with the square of A_0 and therefore the correspondting component of the power amplification factor tends to infinite when the output voltage tends to zero.

The value of A_0 itself may be reduced, say, by slightly overcompensating the secondary but instability becomes then almost certain.

The component of the power amplification factor corresponding to ampereturns A_r is, on the contrary tending to zero when the load decreases and the chosen law of amplification is represented by equation (1). The influence of the other terms, A_s and A_q may be similarly commented.

The influence of speed variations upon the fidelity to law (1) depends chiefly on the ratio $\frac{A_r}{A_0}$; the higher this ratio the lesser the influence. Fig. 51 represents this influence for a unit tested at the University of Minnesota; this influence is negligible when speed variations are of the order of a few percent as it is the usual case.

Instead of the law (1) corresponding to an horizontal characteristic $a\,b\,c$ shown in fig. 52 we may chose following one

$$I_{bd} = k_\gamma I_\gamma + k_\delta I_\delta + k_\varepsilon I_\varepsilon + k_x V_x \tag{6}$$

where V_x is the voltage induced by the consumer, and in particular, the difference of potential E_{BD} between the secondary terminals of the metadyne; $d\,b\,e$ or $d\,f\,g$ are such characteristics.

The arrangement of the stator winding, the fidelity with which law (6) is followed and the value of the power amplification factor, may be commented in a similar manner as here above developed in connection with law (1).

Fig. 53 shows such characteristics obtained at the laboratory of the University of Minnesota.

Let us now consider following law:

$$E_{BD} = k_\gamma I_\gamma + k_\delta I_\delta + k_\varepsilon I_\varepsilon \tag{7}$$

represented by the vertical straight line $h\,k$ in fig. 52.

We may write equations (2) as follows:

$$\begin{aligned} R_\pi I_{ac} + n K_\sigma I_{bd} &= \Sigma n K_{ac}^{\delta} I_\delta + n K_{ac}^{\zeta} \frac{E_{BD}}{R_\zeta} \\ n K_\pi I_{ac} - R_\sigma' I_{bd} &= + E_{BD} \left(1 + n K_{bd}^{\zeta} \frac{1}{R_\zeta} \right) \end{aligned} \tag{8}$$

where R_σ' comprises ohmic and kinetic resistance inside

(1) The figures 50, 51, 53, 54, 55, 56, 57, 58, 59 are non-existent on the original writing.

the metadyne. Solving for I_{ac} and E_{BD}, we obtain:

$$
(9)\qquad
\begin{aligned}
I_{ac} &= \frac{\left(n K_\sigma I_{bd} - \Sigma\, n K_{ac}^{\delta} I\delta\right)\left(1 + n K_{bd}^{\zeta}\dfrac{1}{R_\zeta}\right) + R_\sigma' I_{bd}\, n K_{ac}^{\zeta}\dfrac{1}{R_\zeta}}{n^2 K_\pi K_{ac}^{\zeta}\dfrac{1}{R_\zeta} - R_\pi\left(1 + n K_{bd}^{\zeta}\dfrac{1}{R_\zeta}\right)} \\
E_{BD} &= \frac{\left(R_\pi R_\sigma' + n^2 K_\pi K_\sigma\right) I_{bd} - n K_\pi \Sigma\, n K_{ac}^{\delta} I\delta}{n^2 K_\pi K_{ac}^{\zeta}\dfrac{1}{R_\zeta} - R_\pi\left(1 + n K_{bd}^{\zeta}\dfrac{1}{R_\zeta}\right)}.
\end{aligned}
$$

If a voltage E_{BD} independent of the load is desired, then following equation must be satisfied:

$$(10)\qquad R_\pi R_\sigma' + n^2 K_\pi K_\sigma = 0.$$

We may instance chose:

$$(11)\qquad R_\sigma' = 0; \qquad K_\sigma = 0$$

and then I_{ac} will also be independent of the load.

Les us connect the shunt windings 4 and 5 across two points of the external circuit between which the kinetic e.m.f. V_x is induced without appreciable ohmic drop. Then conditions (11) become:

$$(12)\qquad R_\sigma = 0; \qquad K_\sigma = 0$$

and the voltage V_x becomes independent of the load but stability is then precarious.

Formula (9) shows how we may obtain any desired linear characteristic referred to same system of coordinates as in fig. 52, but inclined with respect the vertical.

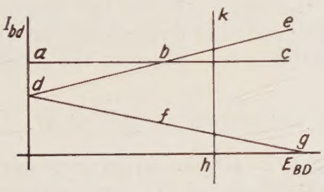


Fig. 52.

Same formula shows that speed variations would effect the voltage E_{BD} with practically the same error in percentage, if the shunt windings and the primary stabilizer are omitted.

Let us now turn our attention to variations respect time.

The schema of fig. 49 is investigated in many sections of the previous Chapters, and a variety of means for improving the stability of the operation and for increasing the rapidity of response are there indicated. Comparing these means with the requirements here above indicated for increasing the value of the « power amplification factor » and for reducing the value of the « distorsion factor » we find most of them divergent; in other words most means enhancing stability and quick response, hamper power amplification, and fidelity.

The device using auxiliary amplifier metadynes for energizing the stator windings of the main metadyne is a mean enhancing both stability and quick response if properly used.

The reason why means improving quick response, reduce power amplification may be easily visualized arguing as follows:

A.) In order to increase quick response the pilot current causes the rapid surge of comparatively high values of e.m.f. tending to establish the desired asymptotic value of the output currents and therefore the power involved in the pilot circuit increases generally with the square of the induced e.m.f. and hence the power amplification factor decreases with the same law.

B.) The frontier variety separating the stability domain from the domain of instability without oscillations separates points where with a small original power causes a definite output from point where same small original power causes an output tending to infinite. Let us arrange for the pilot current to supply this original power and for the representative point to remain within the stability domain and approach the here above mentioned frontier variety. The more we approach said frontier variety, the higher become the value of the power amplification factor, but also the greater the danger that a small cause will shift to instability.

The device using auxiliary amplifier metadynes is favorable to both increase of power amplification factor and quick response because it introduces an indirect increase of power due to transformation of mechanical power to electrical power performed by the rapid rotation of the amplifier metadynes.

In « Metadyne Periodics » a complete chapter is assigned to the construction of graphic characteristic recording the operation of an S generator metadynes under alternating current of variable pulsation. Such characteristics will be considered further in « Metadyne Combinatory », in connection with cybernetics.

Fig. 54 and 55 show series of S amplifier metadynes constructed for standard application and the graphic representation of the corresponding transfer function. Similar are fig. 56 and 57 relative to a 20 kW system for a simulator used in the control of guided missiles. Fig. 58 and 59 refer to the metadyne laboratory of the University of Minnesota.

In this section we have, so far, silently admitted that the motor driving the $S\,G\,M$ is able to rotate at a constant speed developing any torque required by the amplifier metadyne. Now we will assume that the driving torque is constant and that the amplifier metadyne is further provided with a regulator dynamo. We may, for instance follow, the general schema shown by fig. 60 where the $S\,A\,M$ is shown driven by a compensated, dynamo M energized by a metadyne source, generator or transformer, and controlled by the excitation windings 11, 12, 13. The amplifier metadyne comprises any series stator winding, the already considered secondary variator windings 1, 2, 3, the shunt windings, 4, 5, and a primary regulator winding 6 traversed by regulator current I_ρ supplied by regulator dynamo $R\,D$ connected to a battery $B\,T$.

The current, Y_{bd}, supplied by the metadyne source is controlled by secondary variator winding 14, 15, 16.

No ausiliary amplifier metadynes are shown by fig. 60 for an easier inspection of the schema, but they are assumed to be operating.

If the speed n is very closely regulated as to admit it constant the static equations are following:

$$R_\pi I_{ac} + n K_\sigma I_{bd} = \Sigma\, n K^{\delta}_{ac} I\delta + n K^{\zeta}_{ac} \frac{V_{BD}}{R_\zeta}$$

$$(13) \qquad n K_\pi I_{ac} - R_\sigma I_{bd} = - V_x - n K^{\zeta}_{bd} \frac{V_{BD}}{R_\zeta} - n K^{\rho}_{bd} I_\rho$$

$$V_{ED} I_{bd} + R_p I^2_{ac} + R'_s I^2_{bd} + P_e = n k (i_1 + i_2 + i_3)(Y_\alpha + Y_\beta + Y_\gamma)$$

where V_{BD} is the voltage between terminals B and D; R_s' is the ohmic resistance of the secondary circuit inside the amplifier metadyne; P_e are losses due to friction, iron hysteresis, shunt windings; i_1, i_2, i_3 are

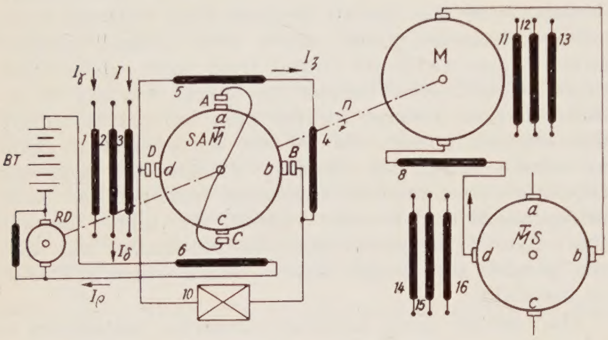


Fig. 60.

the currents traversing windings 11, 12, 13 and Y_α, Y_β, Y_γ, the currents traversing windings 14, 15, 16.

If we neglect terms $R_p I^2_{ac}$, and P_e as very small with respect the other, the third of equation (13) yields readily the output voltage current characteristic.

Term $R'_s I^2_{bd}$ is generally small the other two remaining terms and may neglected; the characteristics are then equilateral hyperbolas when the product $(i_1 + i_2 + i_3)$ $(Y_\alpha + Y_\beta + Y_\gamma)$ is constant, as shown in fig. 61. If term

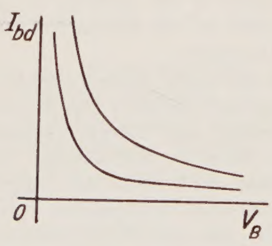


Fig. 61.

$R'_s I^2_{bd}$ cannot be neglected then one of the asymptotes does not coincide with the coordinate axes. Currents i_1, i_2, i_2 and Y_α, Y_β, Y_γ may then be considered as pilot currents and the two first equations (13) determine I_{ac} and I_ρ as functions of currents I_γ, I_δ, I_ε.

We have here an amplification which follows a non linear law.

Let us assume that the consumer 10, comprises a resistor of resistance R_x and a machine inducing the electromotive force V_x and having a negligible internal ohmic drop; further that windings 12 and 15 are shunt connected across said machine of the consumer; that windings 11 and 14 are traversed by arbitrarily predetermined currents, and that windings 13 and 16 are traversed by I_{bd}. Then the third equation (13) may be written as follows, if terms $R_p I^2_{ac}$ and P_e are neglected:

$$V_x I_{bd} + R_s I^2_{bd} = n k (a V_x + b I_{bd} + c) \cdot (a' V_x + b' I_{bd} + c')$$

and by ordering terms:

$$(14) \qquad 0 = a_{11} V_x^2 + 2 a_{12} V_x I_{bd} + 2 a_{22} I^2_{bd} + 2 a_{13} V_x + a_{21} I_{bd} + a_{33}$$

where

$$(15) \qquad \begin{aligned} a_{11} &= n k a a'; \qquad 2 a_{12} = -1 + (a b' + a' b) n k; \\ 2 a_{22} &= n k b b' - R_s; \\ 2 a_{13} &= n k (a c' + a' c); \qquad 2 a_{23} = n k (b c' + b' c); \\ a_{33} &= n k c c'; \end{aligned}$$

We dispose of 5 parameters, say a, b, c, $\frac{a'}{c'}$, $\frac{b'}{c'}$ and through them we may determine all five ratios of the coefficients of equation (14):

$\frac{a_{11}}{a_{33}}$, $\frac{a_{12}}{a_{33}}$, $\frac{a_{22}}{a_{33}}$, $\frac{a_{13}}{a_{33}}$ and $\frac{a_{23}}{a_{33}}$ and hence give to the conic represented by equation (14) any desired form. If the coefficients of all linear terms are eliminated the center of the conic lies on the origin of the coordinate axes. We may have hyperbolas, parabolas, ellipses, and circles located at any place of the axes.

4.) Amplifier metadyne type CAM.

Fig. 62 shows the schema of an amplifier type CAM; the primary terminals, A, C, are connected to a d.c. network of voltage, V_x; the secondary terminals B, D, are connected to a consumer. The amplifier metadyne operates simultaneously as transformer and as a gene-

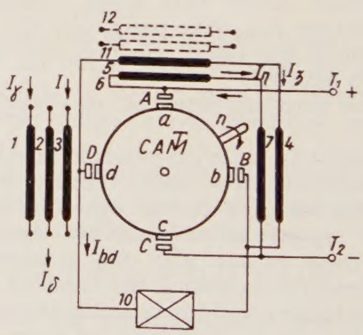


Fig. 62.

rator driven by a motor at a constant speed n. There are any series windings and further three pilot windings 1, 2, 3 and four shunt windings, 4 and 5 connected to the secondary terminals 6 and 7 connected to the primary terminals. The consumer 10 is characterized by resistance R_y and e.m.f. V_y.

The static equations may be written as follows:

$$(16)\qquad \begin{aligned} R_\pi I_{ac} + n K_\sigma I_{bd} &= V_x + \Sigma n K_{ac}^{\delta} I_\delta + \\ &+ n K_{ac}^{\eta} \frac{V_x}{R_\eta} + n K_{ac}^{\zeta} \frac{R_y I_{bd} + V_y}{R_\zeta} \\ n K_\pi I_{ac} - R_\sigma I_{bd} &= \\ &= - V_y - n K_{bd}^{\eta} \frac{V_x}{R_\eta} - n K_{bd}^{\zeta} \frac{R_y I_{bd} + V_y}{R_\zeta} \end{aligned}$$

Solving equations (16) for I_{ac} and I_{bd}, we obtain:

$$(17)\qquad I_{bd} = \frac{n K_\pi \Sigma n K_{ac}^{\delta} I_\delta + a V_x + b V_y}{R_\pi \left(R_\sigma - n K_{bd}^{\zeta} \frac{R_y}{R_\zeta} \right) + n K_\pi \left(n K_\sigma - n K_{ac}^{\zeta} \frac{R_y}{R_\zeta} \right)}$$

where

$$(18)\qquad \begin{aligned} a &= R_\pi n K_{bd}^{\eta} \frac{1}{R_\eta} + n K_\pi \left(1 + n K_{ac}^{\eta} \frac{1}{R_\eta} \right) \\ b &= R_\pi \left(1 + n K_{bd}^{\zeta} \frac{1}{R_\zeta} \right) + n K_\pi n K_{ac}^{\zeta} \frac{1}{R_\zeta}. \end{aligned}$$

We dispose of the values of four parameters:

$$K_{ac}^{\zeta} \frac{1}{R_\zeta}; \qquad K_{bd}^{\zeta} \frac{1}{R_\zeta}; \qquad K_{bd}^{\eta} \frac{1}{R_\eta}; \qquad \text{and } K_{ac}^{\eta} \frac{1}{R_\eta}$$

hence we may give any desired value to coefficients a and b and thus obtain an output current I_{bd} as any desired linear function of the pilot current and the voltages V_x and V_y. In particular for $a = 0$ and $b = 0$, we have an output current exactly proportional to any linear combination of the pilot currents.

Solving equations (16) for I_{ac} and V_y, we have:

$$V_y = \frac{- n K_\pi \Sigma n K_{ac}^{\delta} I_\delta + c V_x + d I_{bd}}{R_\pi \left(1 + n K_{bd}^{\zeta} \frac{1}{R_\zeta} \right) + n^2 K_\pi K_{ac}^{\zeta} \frac{1}{R_\zeta}}$$

where

$$(19)\qquad \begin{aligned} c &= - R_\pi n K_{bd}^{\eta} \frac{1}{R_\eta} - n K_\pi \left(1 - n K_{ac}^{\eta} \frac{1}{R_\eta} \right) \\ d &= R_\pi \left(R_\sigma - n K_{bd}^{\zeta} \frac{R_y}{R_\zeta} \right) + \\ &+ n K_\pi \left(n K_\sigma - n K_{ac}^{\zeta} \frac{R_y}{R_\zeta} \right). \end{aligned}$$

These formulae show that, by conveniently setting our free parameters, we may obtain a voltage V_y as any desired linear function of the pilot currents, the primary voltage V_x and the output current. In particular if we set $c = 0$ and $d = 0$ the voltage V_y is exactly proportional to any linear combination of the pilot currents.

Thus the *C A M* is reacher in possible combinations than the *S A M* considered in the previous section. Further there is another important advantage of the former amplifier; there are applications of great interest where a high fidelity is required around the zero value of the sum:

$$(20)\qquad \Sigma n K_{ac}^{\delta} I_\delta$$

much higher than for values very different from zero. Yet the *S A M* is practically unmagnetized for values of said sum substantially equal to zero and the remanent magnetism, affects heavily the exactitude of the operation. On the contrary the magnetic circuit of the *C A M* is still magnetized to an extent proportional to the primary voltage V_x when the sum (20) is around its zero value and hence hysteresis affects very little the operation.

It is important also to note that, in case it is desired to have the output current exactly potortional to sum (20), it is not necessary that the primary voltage be constant.

Adding a plurality of primary variator windings, as indicated in fig. 61 by 11 and 12, another way of controlling the output is offered. To the numerator of formula (17) following term is added:

$$R_\pi \Sigma n K_{bd}^{\theta} I_\theta$$

where I_θ is the generic symbol for the pilot currents traversing said primary windings.

To the numerator of the first formula (19) same term is added. The intensity of the action of the secondary pilot current as compared to the action of the primary pilot currents is measured by the ratio

$$\frac{n K_\pi}{R_\pi}.$$

Let us assume now that the primary voltage, V_x, and the primary current, I_{ac}, are correlated by:

$$(21)\qquad \mathcal{S}(V_x, I_{ac}) = 0$$

what will be the output?

To equation (21) we must add the two equations (16) completed by term:

$$(22)\qquad - \Sigma n K_{bd}^{\theta} I_\theta$$

into the second member of the second equation (16). Let us solve the thus completed equations (16) with

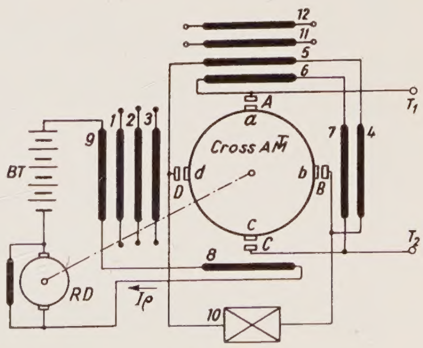


Fig. 63.

respect I_{ac} and V_x and substitute into (21); we will have the equation

$$(23)\qquad f(V_y, I_{bd}) = 0$$

correlating the output current and voltage. If (21) is an albegraic equation of order h, (23) will also be an algebraic equation of same order whose coefficients are linear functions of the pilot currents. As in fact the sums (20) and (22) act as two independent parameters, the $C\ A\ M$ will operate as a transformer of equation (21) to equation (23) by modifying two coefficients. This suffices for instance for transforming the nature of any conic (even a degenerated one into two straight lines); we may thus have, for instance, an hyperbola from an ellipse or a parabola from two straight lines (or a double straight line).

5.) Amplifier transformer metadyne type Cross *T M*.

The schema is shown by fig. 63 similar to fig. 62 except for the addition of a regulator dynamo, RE, and two regulator windings, 8, 9. [2]

[2] Here is ended our acccunt for suspension of original writing, the group of initial four chapters (I, II, III, IV) of Metady Dinamics: «LINEAR TRANSIENTS».

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